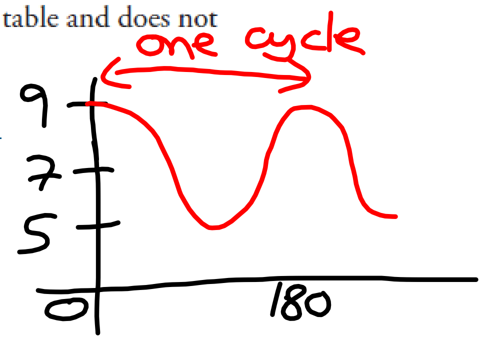
Solutions

2. Determine the function that models the data in the table and does not involve a horizontal translation.

x	0°	45°	90°	135°	180°	225°	270°
y	9	7	5	7	9	7	5



Max = 9 Min = 5

Equation of axis $= \frac{9+5}{2}$

$= \frac{14}{2}$

$= 7$

Amplitude $= \frac{9-5}{2}$

$= \frac{4}{2}$

$= 2$

Starts at a max

$\Rightarrow$ use positive cosine

Period = 180

$\Rightarrow k = \frac{360}{180} = 2$

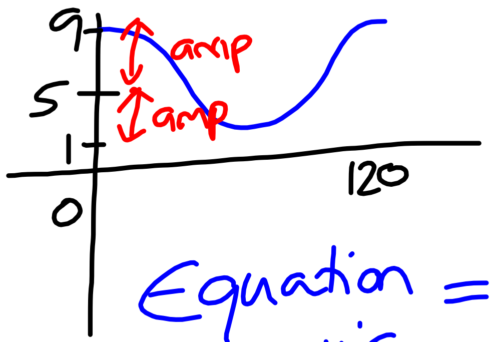
$\Rightarrow y = 2\cos(2x) + 7$

3. A sinusoidal function has an amplitude of 4 units, a period of 120°, and a maximum at (0, 9). Determine the equation of the function.

Amplitude = 4

Period = 120 $\Rightarrow k = \frac{360}{120} = 3$

Max at (0,9) $\Rightarrow$ Starts at a max, so use positive cosine



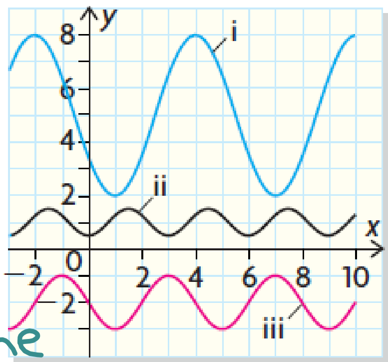
$\Rightarrow y = 4\cos(3x) + 5$

Equation of axis $= \frac{9+1}{2} = \frac{10}{2} = 5$

$\Rightarrow y = 5$

4. Determine the equation for each sinusoidal function.

a)



(i) Max 8, Min 2

Eqn of axis $= \frac{8+2}{2} = 5$

Amplitude $= \frac{8-2}{2} = 3$

If start at a min $\Rightarrow$ − cosine

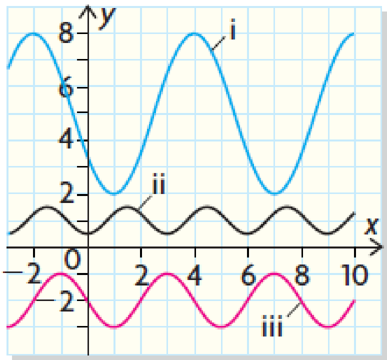
phase shift of right 1 $k = \frac{360}{6} = 60$

$\Rightarrow y = -3\cos(60(x-1)) + 5$

OR $y = 3\cos(60(x-4)) + 5$ if using the first maximum

4. Determine the equation for each sinusoidal function.

a)



(ii) Max = 1.5, Min = 0.5

Eqn of axis $= \frac{1.5+0.5}{2} = 1$

Amplitude $= \frac{1.5-0.5}{2} = \frac{1}{2}$

Starts at a min $\Rightarrow$ − cosine

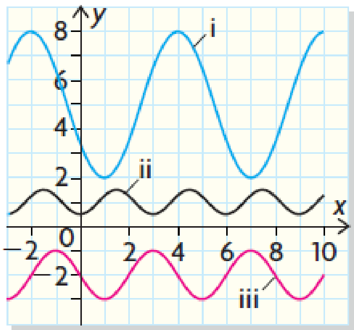
$k = \frac{360}{3} = 120$

$\Rightarrow \quad y = -\frac{1}{2}\cos(120x) + 1$

OR $\quad y = \frac{1}{2}\cos(120(x-1.5)) + 1$ if using the first maximum

4. Determine the equation for each sinusoidal function.

a)



(iii) Max = −1, Min = −3

Eqn of axis $= \frac{-1+-3}{2} = -2$

Amplitude $= \frac{-1-(-3)}{2} = 1$

Starts at the axis and decreases $\Rightarrow$ use − sine

$k = \frac{360}{4} = 90$

$\Rightarrow \quad y = -\sin(90x) - 2$

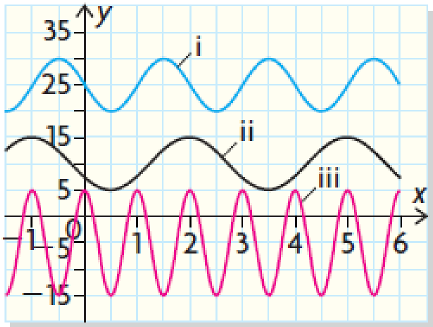
OR $\quad y = \sin(90(x-2)) - 2$

OR $\quad y = \cos(90(x-3)) - 2$

OR $\quad y = -\cos(90(x-1)) - 2$

4. Determine the equation for each sinusoidal function.

b)



(i) Max = 30, Min = 20

Eqn of axis $= \frac{30+20}{2} = 25$

Amplitude $= \frac{30-20}{2} = 5$

Starts on the axis and decreases $\Rightarrow$ use $-$sine

$k = \frac{360}{2} = 180$

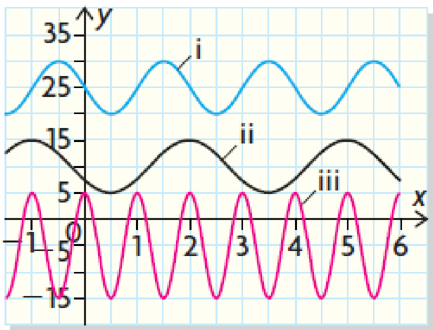
$\Rightarrow y = -5\sin(180x) + 25$

OR $y = -5\cos(180(x-0.5)) + 25$

OR $y = 5\cos(180(x-1.5)) + 25$

4. Determine the equation for each sinusoidal function.

b)



(ii) Max = 15, Min = 5

Eqn of axis $= \frac{15+5}{2} = 10$

Amplitude $= \frac{15-5}{2} = 5$

Using the first minimum $\Rightarrow$ use $-$cosine with a phase shift of right 0.5

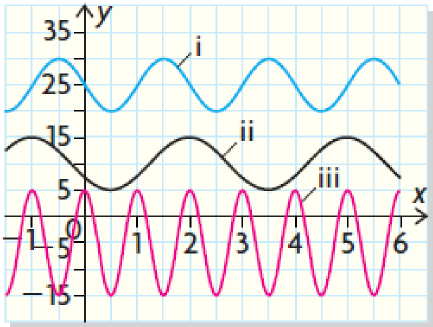
$k = \frac{360}{3} = 120$

$\Rightarrow y = -5\cos(120(x-0.5)) + 10$

OR $y = 5\cos(120(x-2)) + 10$ if using the first maximum

4. Determine the equation for each sinusoidal function.

b)



(iii) Max = 5, Min = −15

Eqn of axis = $\frac{5+(-15)}{2} = -5$

Amplitude = $\frac{5-(-15)}{2} = 10$

$k = \frac{360}{1} = 360$

Starts at a maximum
⇒ use positive cosine

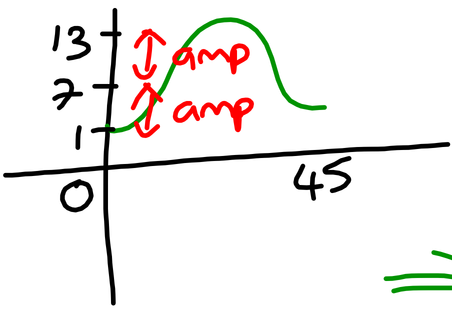
$\Rightarrow y = 10\cos(360x) - 5$

7. A sinusoidal function has an amplitude of 6 units, a period of 45°, and a minimum at (0, 1). Determine an equation of the function.

Amplitude = 6

Period = 45 $\Rightarrow k = \frac{360}{45} = 8$

Starts at a minimum of 1 ⇒ −cos



Eqn of axis = $\frac{13+1}{2} = 7$

$\Rightarrow y = -6\cos(8x) + 7$

8. The table shows the average monthly high temperature for one year in Kapuskasing, Ontario.

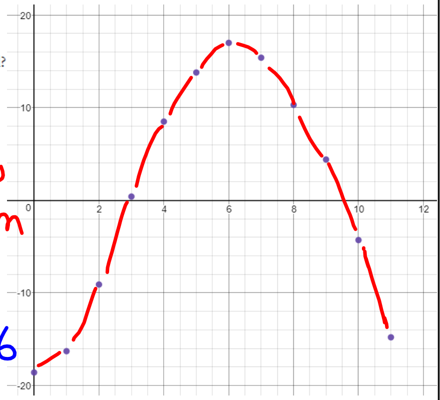
Time (months)	J	F	M	A	M	J	J	A	S	O	N	D
Temperature (°C)	−18.6	−16.3	−9.1	0.4	8.5	13.8	17.0	15.4	10.3	4.4	−4.3	−14.8

a) Draw a scatter plot of the data and the curve of best fit. Let January be month 0.

b) What type of model describes the graph? Why did you select that model?

c) Write an equation for your model. Describe the constants and the variables in the context of this problem.

d) What is the average monthly temperature for month 20?



b) Sinusoidal. Looks to have a repeating pattern every 12 months.

c) Max = 17.0 Min = −18.6

$\text{Eqn of axis} = \dfrac{17.0 + -18.6}{2} = -0.8$

$\text{Amplitude} = \dfrac{17.0 - (-18.6)}{2} = 17.8$

Starts at a minimum $\Rightarrow$ use − cosine

$\Rightarrow f(x) = -17.8\cos(30x) - 0.8$

d) $f(20) = -17.8\cos(30(20)) - 0.8$

$= 8.1°C$