Warm Up:
State the transformations applied to the sine function.

$$f(x) = -2\sin(x - 60) + 3$$

Vertical stretch by a factor of 2
Reflection in the x-axis
Phase shift right 60
Vertical translation up 3

Writing Sine and Cosine Equations

Lesson objectives

- I can take the properties for a periodic function from the graph and write an equation
- I can take the properties of a periodic function from a table of values and write an equation
- I can take the properties of a periodic function from a word problem and write an equation

1.1

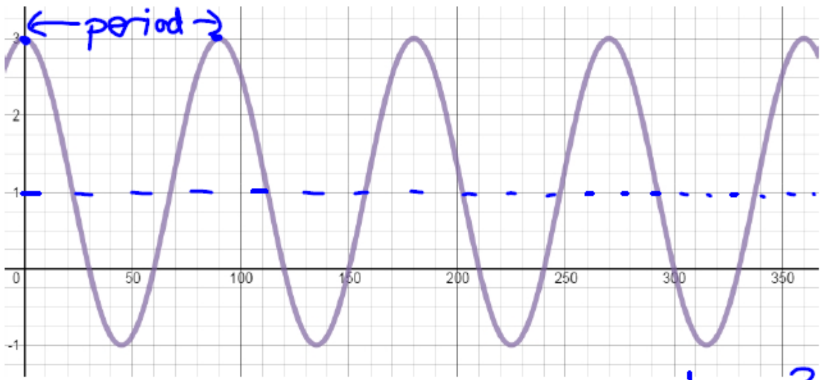
Lesson objectives | Teachers' notes | Lesson notes

Nelson Page 391 #s 2, 3, 4, 7 & 8

Understanding how period, amplitude, maxand min and the axis of curve relate to the equation

Let's examine the following cosine curve.

Describe the transformations to the curve.



Max = 3
Min = -1
Period = 90

Axis of curve $= \frac{max + min}{2} = \frac{3+(-1)}{2}$

$\Rightarrow y = 1$

Amplitude $= \frac{max - min}{2} = \frac{3-(-1)}{2}$
$= 2$

Complete the chart below for each equation:

	y = cos(x)	y = 2cos(4x) + 1
max value	1	3
min value	−1	−1
axis of curve	y = 0	y = 1
amplitude	1	2
period	360°	90°
# of cycles in 360°	1	4
"a"	1	2
"k"	1	4
"d"	0	0
"c"	0	1

Any connections?

$k = \frac{360}{Period}$ $\left[Period = \frac{360}{k}\right]$

Axis of curve $\Rightarrow y = c$

Amplitude $\Rightarrow a$

Max $= c + |a|$
Min $= c - |a|$ } positive "version" of a

Understanding the Symmetry of a Sinusoidal Curve

To be able to model using a sinusoidal curve we need to understand the symmetry to be able to draw out patterns.

Let's examine the sine function.

Important Points

Starts at: (0, 0)

First quarter: (90, 1)

Halfway: (180, 0)

Third quarter: (270, -1)

Ends at: (360, 0)

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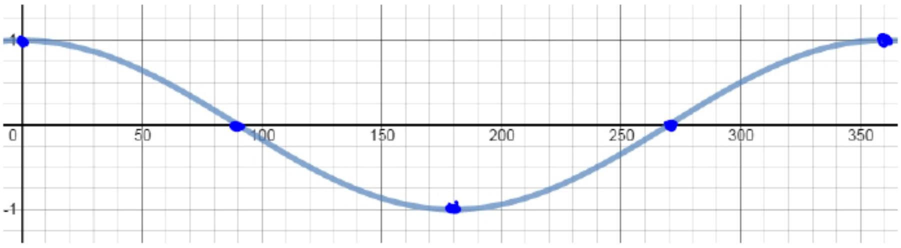

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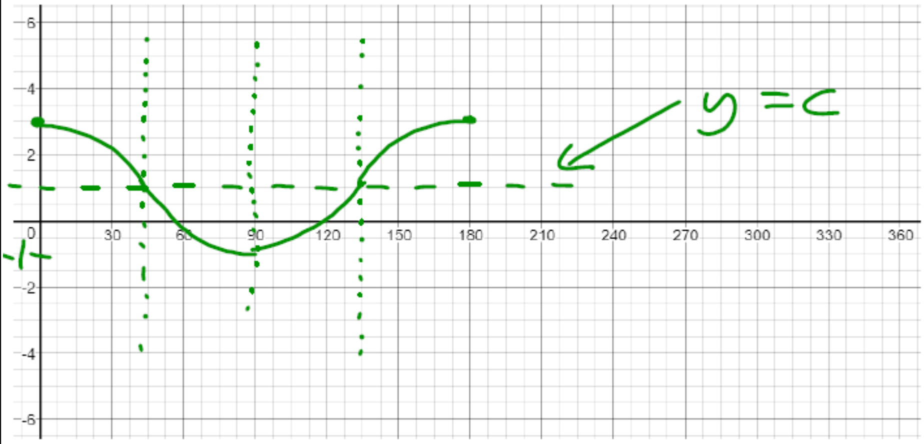
Example

A sinusoidal function has an amplitude of 2 units, a period of 180°, and a max at (0,3). Representthe function with two different equations.

1. Sketch the curve.



2. Which function starts at a max? ⇒ cosine

$$y = 2\cos(2x) + 1$$

Example

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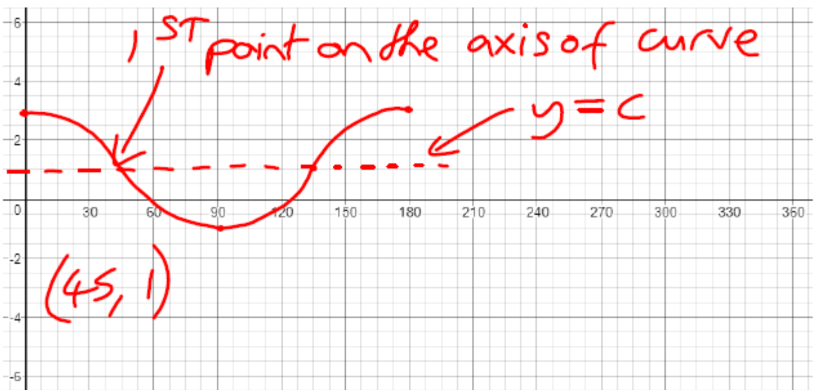


2. Since sine does not start at a max what horizontal shift has taken place?

$$y = -2\sin(2(x-45)) + 1$$

decreases, not increases

phase shift from 0 of right 45

Example

Determine the equation that models this data:

x	0	30	60	90	120	150	180
y	3	2	1	2	3	2	1

What is the max? 3

$k = \frac{360}{\text{period}} = \frac{360}{120} = 3$

What is the min? 1

How long does it take for one cycle? 120 (max → max)

Where is the equation of the axis? $y = 2$

Do we start at the axis or at a max/min? Max ⇒ cos

$\Rightarrow y = \cos(3x) + 2$

Example

Determine the equation that models this data:

x	-180	0	180	360	540	720	900
y	17	13	17	21	17	13	17

What is the max? 21

$k = \frac{360}{720} = \frac{1}{2}$

What is the min? 13

How long does it take for one cycle? 720 (min ⇒ min)

Where is the equation of the axis? $y = 17$

Do we start at the axis or at a max/min? Min ⇒ −cos

$y = -4\cos(0.5x) + 17$

$\text{amp} = \frac{21-13}{2} = 4$

Steps to follow when modelling:

1. Determine the max and min

2. Determine the axis of curve: $y = \dfrac{\text{max} + \text{min}}{2}$

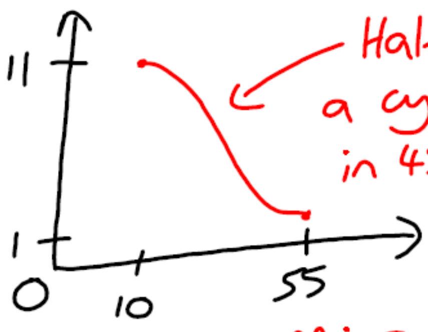
3. Determine the amplitude: $\dfrac{\text{max} - \text{min}}{2}$

4. Determine how many cycles are in 360 to find k.

5. Decide which model
   - Does the graph start at a max/min? ⟶ cosine
   - Does the graph start at the axis? ⟶ sine
   - Neither? ⟶ Your choice - you have to determine the phase shift!

Example

A group of students are tracking their friend, Bunter, on aFerris Wheel. They know that Bunter reaches themaximum height of 11 metres at 10 seconds then reaches the minimum height above the ground of 1 metre at 55 seconds. Find the equation to model Bunter's height above the ground as he travels onthe Ferris Wheel.



Half of a cycle in 45 sec $\Rightarrow$ period $= 2(45)$ $= 90$

$k = \dfrac{360}{90} = 4$

Max $= 11$, Min $= 1$

Axis of curve $= \dfrac{11+1}{2} = 6$

Amplitude $= \dfrac{11-1}{2} = 5$

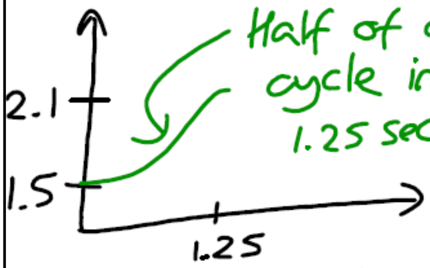
If we use cosine the 1^{ST} max occurs after 10 sec $\Rightarrow d = 10$

$y = 5\cos(4(x-10)) + 6$

Example

Smudger is floating on an inner tube in a wave pool. He is1.5 metres from the bottom of the pool when he is at the trough of the wave. A stop watch starts timing at this point. In 1.25 seconds, he is on the peak of the wave, 2.1 metres from the bottom of the pool.

a) Determine the equation of the function that expresses Smudger's distance from the bottom of the pool in terms of time.



Half of a cycle in 1.25 sec $\Rightarrow$ period $= 2(1.25)$
$= 2.5$

$k = \frac{360}{2.5} = 144$

Max $= 2.1$, Min $= 1.5$

Starts at a min so we can use $-\cos$ with no phase shift

Axis $= \frac{2.1+1.5}{2} = 1.8$

Amplitude $= \frac{2.1-1.5}{2} = 0.3$

$\Rightarrow y = -0.3\cos(144x) + 1.8$

b) How far above the bottom of the pool is Smudger after 4 seconds?

Sub in $x = 4$

$\Rightarrow -0.3\cos(144(4)) + 1.8$

$= 2.04$ m

c) If data was collected for a total of 40 seconds, how many complete cycles would have occurred?

Period $= 2.5$

$\Rightarrow$ # of cycles $= \frac{40}{2.5} = 16$

d) If the period of the function changes to 3 seconds, what is the equation of the new function?

New $k = \frac{360}{3} = 120$

Period affects nothing else

$\Rightarrow y = -0.3\cos(120x) + 1.8$