Transformations of the Sine and Cosine Curves

Lesson objectives

- I can identify a change on the axis of curve/vertical shift
- I can identify a phase shift/horizontal shift
- I can identify a change in amplitude/vertical stretch/compression
- I can identify a change in the period/horizontal stretch/compression
- I can identify a horizontal or vertical reflection

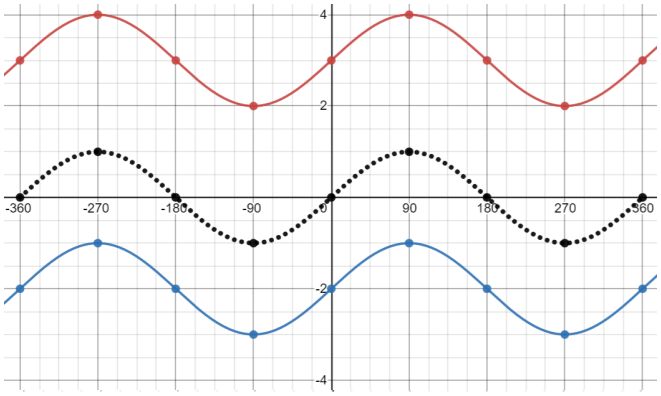
1.1

Lesson objectives | Teachers' notes | Lesson notes

Complete the Handout

1. Complete the following table of values and graph each function in a different colour.

$y = \sin(x) + 3$		$y = \sin(x) - 2$	
x	y	x	y
−360°	3	−360°	-2
−270°	4	−270°	-1
−180°	3	−180°	-2
−90°	2	−90°	-3
0°	3	0°	-2
90°	4	90°	-1
180°	3	180°	-2
270°	2	270°	-3
360°	3	360°	-2



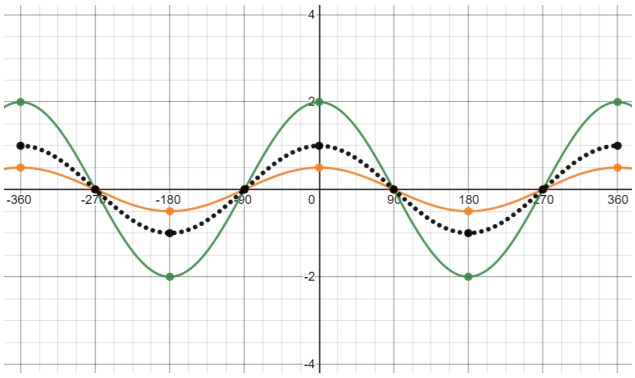
Explain the changes to the base function:

Vertical translations of up 3 and down 2

2. Complete the following table of values and graph each function in a different colour.

$y = 2\cos x$	$y = \frac{1}{2}\cos(x)$

x	y	x	y
−360°	2	−360°	0.5
−270°	0	−270°	0
−180°	-2	−180°	-0.5
−90°	0	−90°	0
0°	2	0°	0.5
90°	0	90°	0
180°	-2	180°	-0.5
270°	0	270°	0
360°	2	360°	0.5



Explain the changes to the base function:

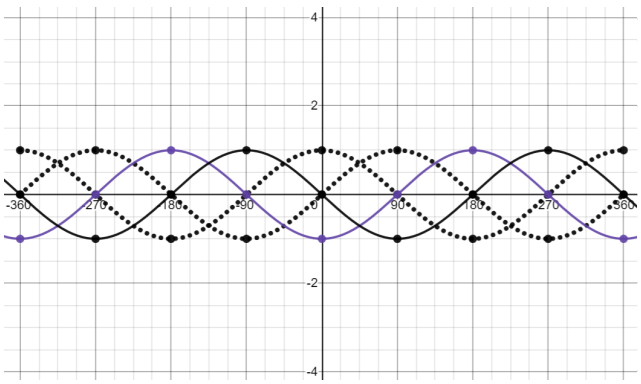
Vertical stretch by a factor of 2

Vertical compression by a factor of 1/2

3. Complete the following table of values and graph each function in a different colour.

$y = -\cos(x)$	$y = -\sin(x)$

x	y	x	y
−360°	-1	−360°	0
−270°	0	−270°	-1
−180°	1	−180°	0
−90°	0	−90°	1
0°	-1	0°	0
90°	0	90°	-1
180°	1	180°	0
270°	0	270°	1
360°	-1	360°	0

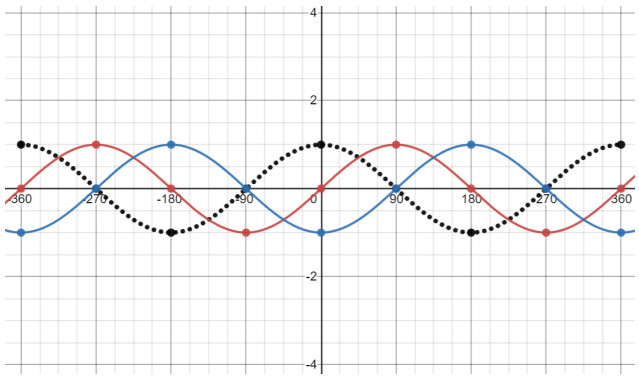


Explain the changes to the base function:

Reflection in the x-axis for both

4. Complete the following table of values and graph each function in a different colour.

$y = \cos(x - 90°)$		$y = \cos(x + 180°)$	
x	y	x	y
−360°	0	−360°	-1
−270°	1	−270°	0
−180°	0	−180°	1
−90°	-1	−90°	0
0°	0	0°	-1
90°	1	90°	0
180°	0	180°	1
270°	-1	270°	0
360°	0	360°	-1

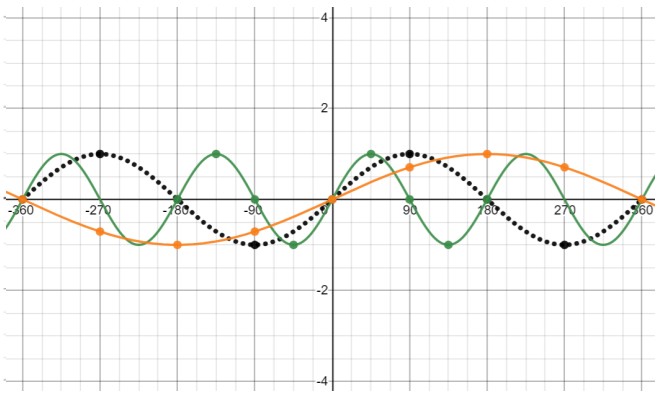


Explain the changes to the base function:

Horizontal translation right 90 and left 180

5. Complete the following table of values and graph each function in a different colour.

$y = \sin(2x)$		$y = \sin\left(\frac{x}{2}\right)$	
x	y	x	y
−180°	0	−360°	0
−135°	1	−270°	-0.707
−90°	0	−180°	-1
−45°	-1	−90°	-0.707
0°	0	0°	0
45°	1	90°	0.707
90°	0	180°	1
135°	-1	270°	0.707
180°	0	360°	0



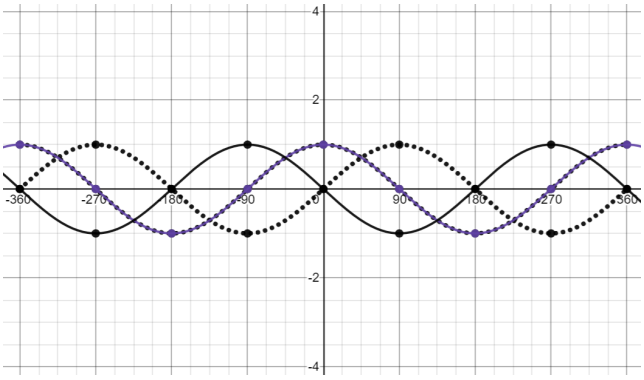
Explain the changes to the base function:

Horizontal compression by a factor of 1/2

Horizontal stretch by a factor of 2

6. Complete the following table of values and graph each function in a different colour.

$y = \cos(-x)$		$y = \sin(-x)$	
x	y	x	y
$-360°$	1	$-360°$	0
$-270°$	0	$-270°$	-1
$-180°$	-1	$-180°$	0
$-90°$	0	$-90°$	1
$0°$	1	$0°$	0
$90°$	0	$90°$	-1
$180°$	-1	$180°$	0
$270°$	0	$270°$	1
$360°$	1	$360°$	0



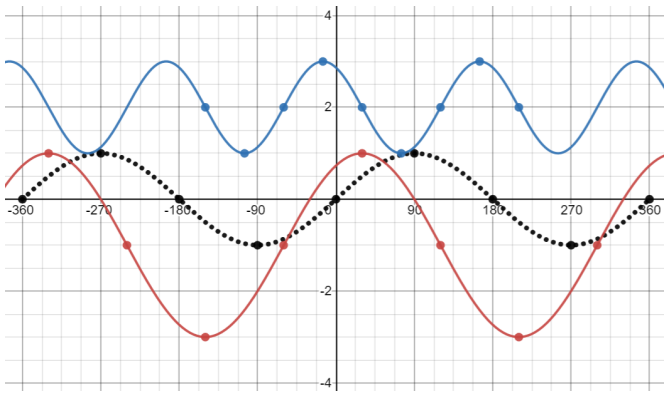
Explain the changes to the base function:

Reflection in the y-axis for both

7. Complete the following table of values and graph each function in a different colour.

$y = 2\sin(x + 60°) - 1$		$y = -\sin(2x - 60°) + 2$	
x	y	x	y
$-330°$	1	$-150°$	2
$-240°$	-1	$-105°$	1
$-150°$	-3	$-60°$	2
$-60°$	-1	$-15°$	3
$30°$	1	$30°$	2
$120°$	-1	$75°$	1
$210°$	-3	$120°$	2
$300°$	-1	$165°$	3
		$210°$	2

Remember to factor to reveal the correct phase shift (HT)



Explain the changes to the base function:

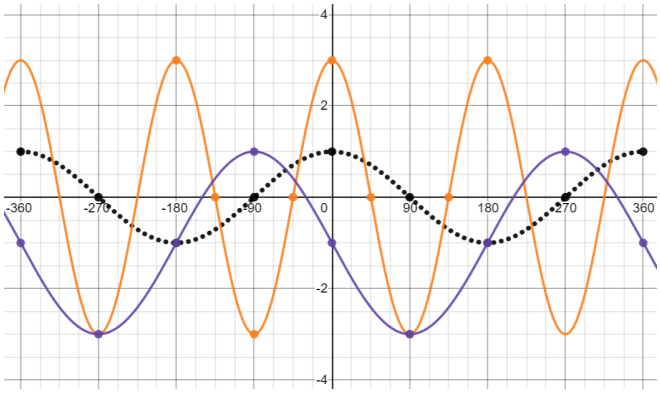
VS by a factor of 2, HT left 60, VT down 1

Reflection in the x-axis, HC by a factor of 1/2, HT right 30, VT up 2

8. Complete the following table of values and graph each function in a different colour.

$y = 3\cos(2x)$	$y = -2\cos(x + 270°) - 1$

x	y	x	y
−180°	3	−360°	-1
−135°	0	−270°	-3
−90°	-3	−180°	-1
−45°	0	−90°	1
0°	3	0°	-1
45°	0	90°	-3
90°	-3	180°	-1
135°	0	270°	1
180°	3	360°	-1



Explain the changes to the base function:
VS by a factor of 3, HC by a factor of 1/2
Reflection in the x-axis, VS by a factor of 2, HT left 270, VT down 1

To summarise....

Trigonometric transformations follow the same rules as for polynomial functions

y = asin(k(x - d)) + c and y = acos(k(x - d)) + c

where

a = vertical stretch/compression/reflection in x-axis
k = horizontal stretch/compression/reflection in y-axis
d = horizontal translation (known as a PHASE SHIFT for trig functions)
c = vertical translation

REMEMBER - To express "d" correctly, we must factor out the value of k if necessary

For example y = 3sin(4x + 80) - 1

y = 3sin(4(x + 20)) - 1 ⟶ d = -20, NOT -80

The key points on a sine or cosine curve are where
x = 0, 90, 180, 270 and 360

We can perform our transformations to each of these key points to see what one transformed cycle will now look like.

Horizontal transformations:

x-value → ÷ k → + d → New x-value

Vertical transformations:

y-value → x a → + c → New y-value