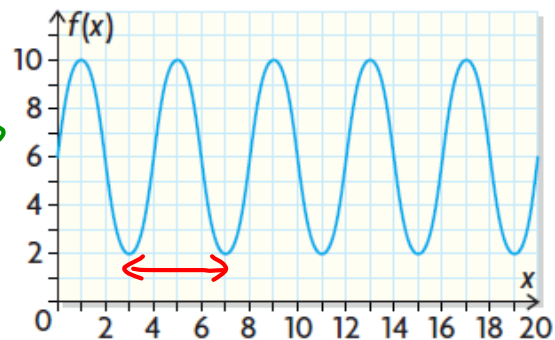# Solutions

2. Determine the range, period, equation of the axis, and amplitude of the function shown.



Range =

$\{f(x) \in \mathbb{R} \mid 2 \leq f(x) \leq 10\}$

Period = length of one cycle.

$= 7 - 3 = 4$

Equation of axis $= \dfrac{max + min}{2}$

$= \dfrac{10 + 2}{2}$

$y = 6$

Amp $= \dfrac{max - min}{2}$

$= \dfrac{10 - 2}{2}$

$= \dfrac{8}{2}$

$= 4$

3. The motion of an automated device for attaching bolts to a household appliance on an assembly line can be modelled by the graph shown at the left.
   a) What is the period of one complete cycle?
   b) What is the maximum distance between the device and the appliance?
   c) What is the range of this function?
   d) If the device can run for five complete cycles only before it must be turned off, determine the domain of the function.
   e) Determine the equation of the axis.
   f) Determine the amplitude.
   g) There are several parts to each complete cycle of the graph. Explain what each part could mean in the context of "attaching the bolt."

**Bolt Machine Motion**
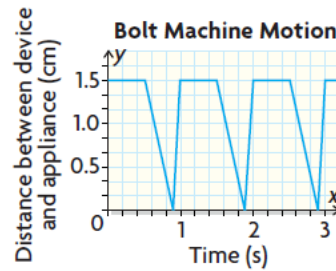
a) one second

b) 1.5 cm

c) $R = \{y \in \mathbb{R} \mid 0 \le y \le 1.5\}$

d) $D = \{x \in \mathbb{R} \mid 0 \le x \le 5\}$

e) Equation of axis $= \dfrac{max + min}{2} = \dfrac{1.5 + 0}{2} \Rightarrow y = 0.75$

f) amplitude $= \dfrac{max - min}{2} = \dfrac{1.5 - 0}{2} = 0.75$ cm

g) 0.5 sec waiting
   0.4 sec bolt approaches appliance
   0.1 sec returns to resting height

---

6. Which of the tables of values might represent periodic functions? Justify.

a)

| x | y |
|---|---|
| −5 | 9 |
| −4 | 4 |
| −3 | 1 |
| −2 | 0 |
| −1 | 1 |
| 0 | 4 |
| 1 | 9 |
| 2 | 16 |

No.
As x increases, y decreases, then increases.
It looks quadratic

b)

| x | y |
|---|---|
| 0.7 | 5 |
| 0.9 | 6 |
| 1.1 | 7 |
| 1.3 | 5 |
| 1.5 | 6 |
| 1.7 | 7 |
| 1.9 | 5 |
| 2.1 | 6 |

Yes.
As x increases, y increases from 5 → 6 → 7.
It then goes back to 5 → 6 → 7.

**6.** Which of the tables of values might represent periodic functions? Justify.

**c)**

| x | y |
|----|------|
| 23 | −6 |
| 26 | −6.5 |
| 29 | −7 |
| 32 | −7.5 |
| 35 | −8 |
| 38 | −8.5 |
| 41 | −9 |
| 44 | −9.5 |

No.
As x increases, y decreases.
It looks linear.

**d)**

| x | y |
|----|---|
| 1 | 5 |
| 2 | 6 |
| 4 | 5 |
| 7 | 6 |
| 11 | 5 |
| 16 | 6 |
| 22 | 5 |
| 29 | 6 |

No.
As x increases, y increases to 6, decreases to 5, increases to 6......

BUT

x is not increasing by the same amount each time.

**7.** Chantelle has a submersible pump in her basement. During a heavy rain, the pump turned off and on to drain water collecting under her house's foundation. The graph models the depth of the water below her basement floor in terms of time. The depth of the water decreased when the pump was on and increased when the pump was off.

Depth of Water under Basement Floor
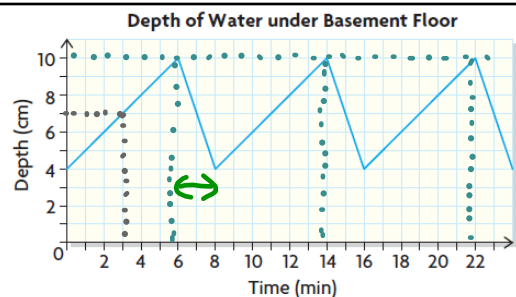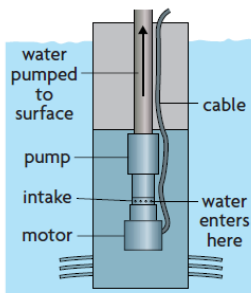
a) Is the function periodic?
b) At what depth does the pump turn on?
c) How long does the pump remain on?
d) What is the period of the function? Include the units of measure.
e) What is the range of the function?
f) What will the depth of the water be at 3 min?
g) When will the depth of the water be 10 cm?
h) What will the depth of the water be at 62 min?

a) Yes.

b) 10cm.

c) 2 mins.

d) 8 mins.

e) $R = \{y \in \mathbb{R} \mid 4 \le y \le 10\}$

f) 7cm

g) 6 mins, 14 mins, 22 mins

h) 56 mins = 7 cycles
62 − 56 = 6 min
Depth at 6 mins = 10cm

3

**8.** While riding on a Ferris wheel, Trevor's height above the ground in terms of
**A** time can be represented by the graph shown.

a) What is the period of this function, and what does it represent?
b) What is the equation of the axis?
c) What is the amplitude?
d) What is the range of the function?
e) After 24 s, when will Trevor be at the lowest height again?
f) At what times is Trevor at the top of the wheel?
g) When will his height be 4 m between 24 s and 30 s?

**Trevor's Height above the Ground**
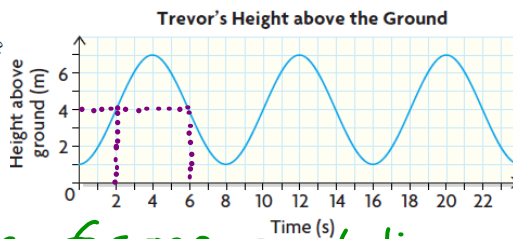
a) 8 seconds. Time taken for one revolution.

b) $y = \dfrac{7+1}{2} \Rightarrow y = 4$

c) amp $= \dfrac{7-1}{2} = \dfrac{6}{2} = 3$

d) $R = \{ y \in \mathbb{R} \mid 1 \le y \le 7 \}$

e) 32 seconds. Min height every 8 seconds

f) 4 sec, 12 sec, 20 sec

g) $24 + 2 = 26$ sec and $24 + 6 = 30$ sec

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**12.** A spacecraft is in an elliptical orbit around Earth. The spacecraft's distance above Earth's surface in terms of time is recorded in the table.

| Time (min) | 0 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 | 78 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Distance (km) | 550 | 869 | 1000 | 869 | 550 | 232 | 100 | 232 | 550 | 869 | 1000 | 869 | 550 | 232 |

a) Plot the data, and draw the resulting curve.
b) Is the graph periodic?
c) What is the period of the function, and what does it represent?
d) What is the approximate distance between the spacecraft and Earth at 8 min?
e) At what times is the spacecraft farthest from Earth?
f) If the spacecraft completes only six orbits before descending to Earth, what is the domain of the function?
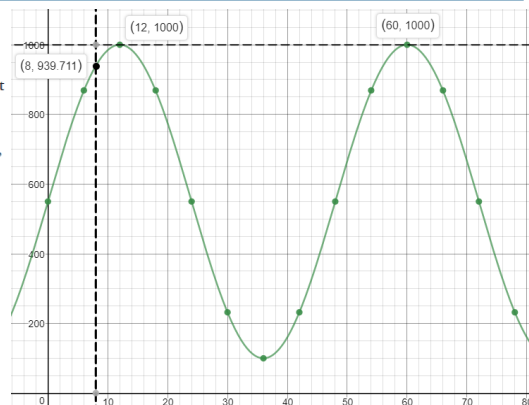
b) Yes.

c) $60 - 12 = 48$ mins
Time for one orbit

d) 940 km

e) 12 mins and 60 mins (and 48 mins after)

f) $D = \{ x \in \mathbb{R} \mid 0 \le x \le 288 \}$

$6 \times 48$ mins $= 288$