Review

1. Topics:

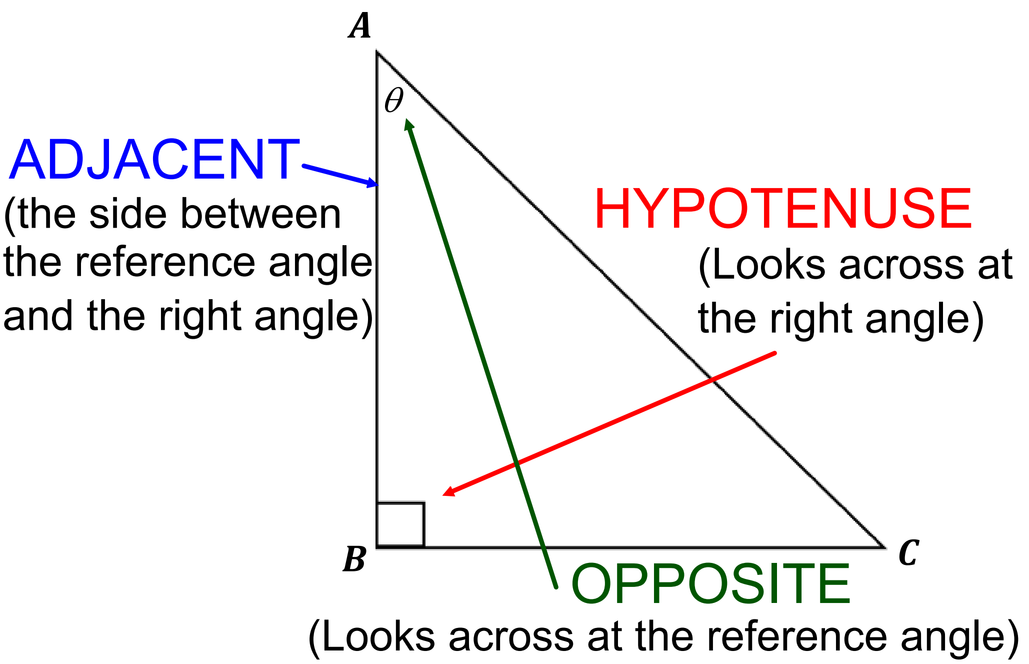
- Primary Trig Ratios (SOH CAH TOA)
- Reciprocal Trig Ratios
- Special Angles
- Angle Terminology (Principal, Negative, RAA etc.)
- Unit Circle
- Solving Trig Equations for angles between 0 and 360
- Trig Identities
- Sine Law and Cosine Law
- The Ambiguous Case
- 3D Trig Problems

2. Review Questions

Nelson Page 338 #s 1 – 4, 7acd, 9, 11 & 12

Labelling a Triangle

Given ΔABC, and using $\angle A$ as the reference angle, label the sides.



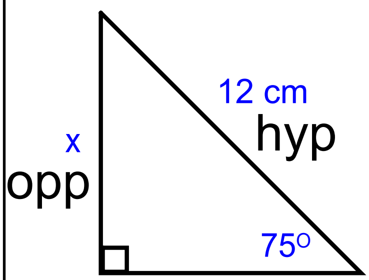
Primary Trig Ratios - SOHCAHTOA

There are three of these with the following formulas:

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

where θ is the measure of the reference angle in the question.

Example: Determine the length of the missing side



1. Label your sides
2. Fill in
   - Have:
   - Need:
   - Use:
3. Sub and solve!

Have: angle, hyp

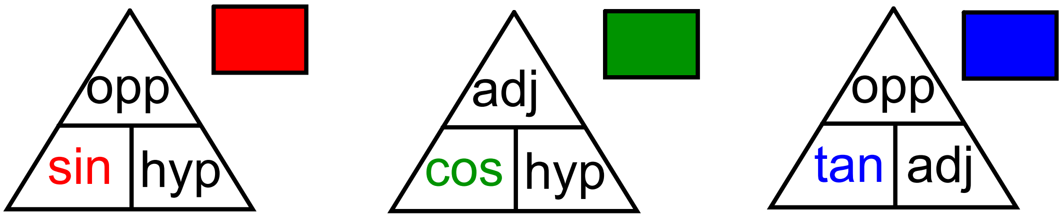
Need: opp

Use: $\sin\theta = \frac{opp}{hyp}$

$\sin(75) = \frac{x}{12}$

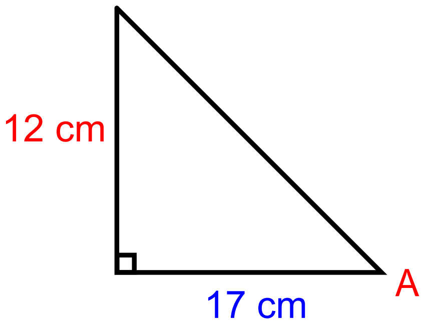
$12\sin(75) = x$

11.6 cm = x



Solving for an Angle

When solving for an angle we must remember that what we find is the ratio, not the angle. To find the angle we must use the reverse lookup by pressing INV, SHIFT, or 2nd on the calculator with the appropriate trig ratio.



Have: opp, adj

Need: angle A

Use: tan

$\tan\theta = \frac{opp}{adj}$

$\tan(A) = \frac{12}{17}$

$A = \tan^{-1}(12 \div 17)$

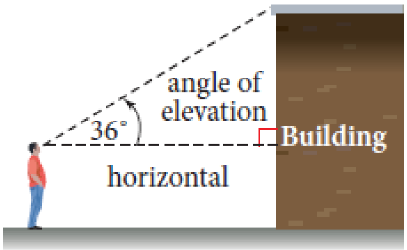
$A = 35.2^\circ$

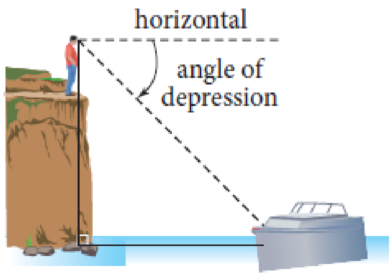
Word Problems

The key to solving word problems is to have a good diagram!

1. Sketch a diagram (include measurements)
2. Label the three sides and determine the reference angle
3. Choose the appropriate ratio using (Have/Need/Use).
4. Determine your missing information.
5. Write a concluding sentence.

When talking about angles, we need to have a reference point. Sometimes, we use an **angle of elevation (inclination)** or an **angle of depression (declination)**.





Reciprocal Trigonometric Ratios

We can flip the primary trig ratios that we know to give us the reciprocal trig ratios. This is most useful in a presentation sense, but not as useful in calculations.

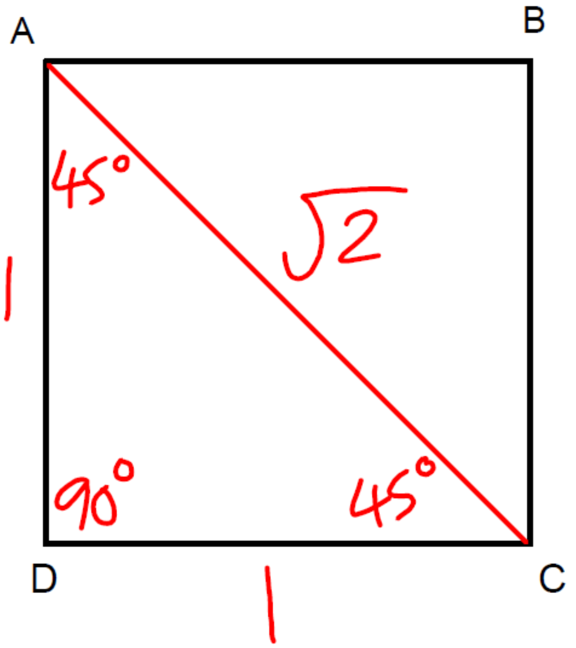
$$\csc\theta = \frac{1}{\sin\theta} = \frac{hyp}{opp} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{hyp}{adj} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{adj}{opp}$$

Co**s**ecant　　Se**c**ant　　Co**t**angent

The **third letter** of each reciprocal ratio links to the ratio that you are flipping.

Special Triangle # 1

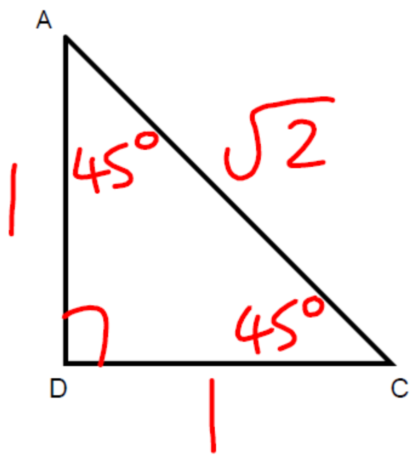
Our first special triangle come from a square with side length = 1.



We can cut this square into two congruent right angle triangles. Because the triangles are congruent angle A and C are split exactly in half.

Special Triangle # 1

If we look at one triangle that we created we will get 3 exact values:



Special Angles:

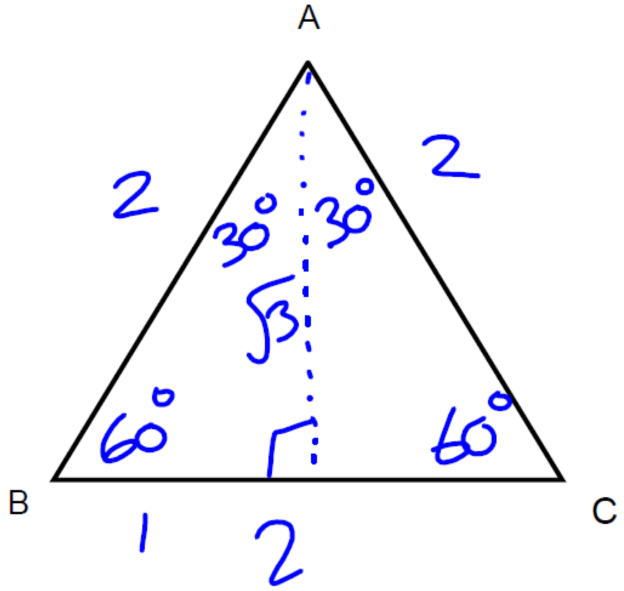
sin 45° = $\frac{1}{\sqrt{2}}$

cos 45° = $\frac{1}{\sqrt{2}}$

tan 45° = 1

Special Triangle # 2

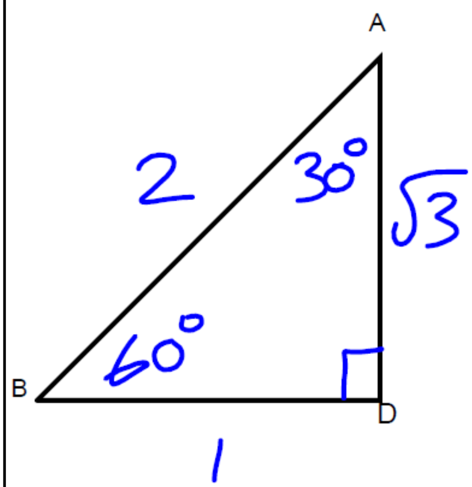
Our second special triangle come from an equilateral triangle with side length = 2.



We can cut this triangle into two congruent right angle triangles. Because the triangles are congruent angle A is split exactly in half.

Special Triangle # 2

If we look at one triangle we will get 2 sets of exact values.



Special Angles:

$\sin 30^\circ = \frac{1}{2}$ $\sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$ $\cos 60^\circ = \frac{1}{2}$

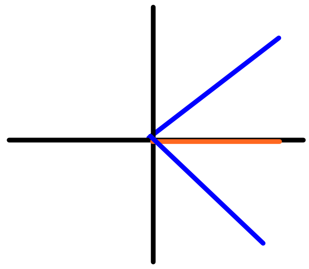
$\tan 30^\circ = \frac{1}{\sqrt{3}}$ $\tan 60^\circ = \sqrt{3}$

$$\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Angle Terminology

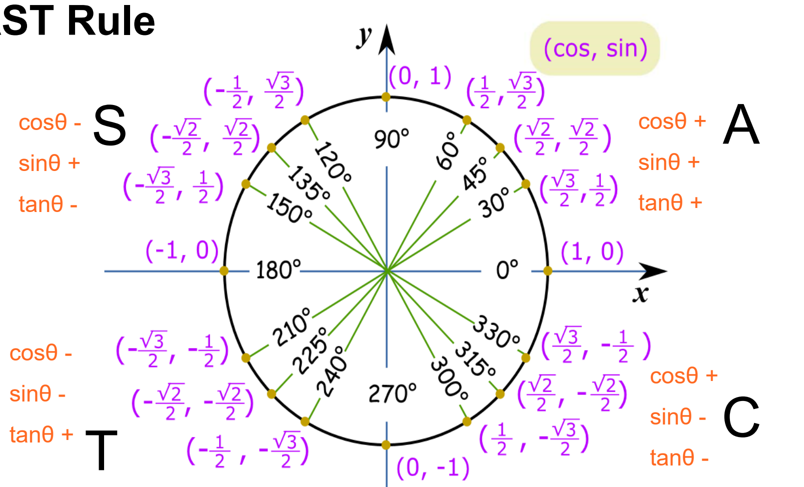
We are going to base angles on the Cartesian Plane.

From here we will define the following:



- Principal Angle
- Coterminal Angle
- Positive vs Negative Angle Measurements
- Related Acute Angles

CAST Rule



We use the letters C.A.S.T. to tell us what is positive in the given quadrant

C - cos   A - all   S - sin   T - tan

Where the ratio is not positive it is negative!

We can summarize the positive/negative values using the C.A.S.T. Rule.
We are talking about the **value of the ratio** not the angle!

Warm Up:

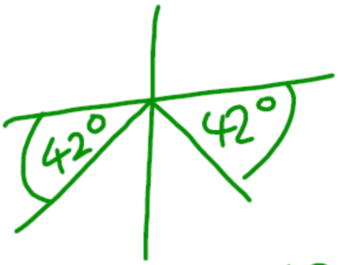
Find all solutions for θ, where $0 \leq \theta \leq 360$.

a) $\sin\theta = -0.6723$ b) $\cos\theta = 0.4291$ c) $\tan\theta = -1.237$

a)

$\theta = \sin^{-1}(-0.6723)$

$\theta = -42°$



$\theta = 180 + 42$

$= 222°$

$\theta = 360 - 42$

$= 318°$

b)

$\theta = \cos^{-1}(0.4291)$

$\theta = 65°$

$\theta = 65°$

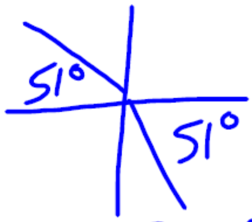
$\theta = 360 - 65$

$\theta = 295°$

c)

$\theta = \tan^{-1}(-1.237)$

$\theta = -51°$



$\theta = 180 - 51$

$= 129°$

$\theta = 360 - 51$

$= 309°$

Trigonometric Identities

Identity: a mathematical statement that is true for all values of the given variable. If the identity involves fractions, the denominators cannot be zero. Any restrictions on a variable must be stated.

- Identities are like puzzles: we have to rearrange equations until they match
- We set up identities in the form LS | RS (left side, right side) and cannot move terms or factors from one side to the other.
- We have 2 main identities from which many others can be formed.

1. Quotient Identity (Q.I.):

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

2. Pythagorean Identity (P.I.):

$$\sin^2\theta + \cos^2\theta = 1$$

Pythagorean Identity

We can rearrange the Pythagorean Identity to get two slightly different identities:

$\sin^2\theta + \cos^2\theta = 1$

1. $\sin^2\theta + \cos^2\theta - \sin^2\theta = 1 - \sin^2\theta$ (subtract $\sin^2\theta$ from both sides)

$\cos^2\theta = 1 - \sin^2\theta$

2. $\sin^2\theta + \cos^2\theta - \cos^2\theta = 1 - \cos^2\theta$ (subtract $\cos^2\theta$ from both sides)

$\sin^2\theta = 1 - \cos^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} \qquad \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta \qquad \tan^2\theta + 1 = \sec^2\theta$$

Reciprocal Identities

Recall: we also have our reciprocal identities:

$\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

Simplify the following expressions:

a) $\cos\theta\tan\theta$

$= \dfrac{\cos\theta}{1} \times \dfrac{\sin\theta}{\cos\theta}$

$= \sin\theta$

where $\cos\theta \neq 0$

b) $\cot\theta\sin\theta$

$= \dfrac{\cos\theta}{\sin\theta} \times \dfrac{\sin\theta}{1}$

$= \cos\theta$

where $\sin\theta \neq 0$

Prove the following identities

a) $\tan\theta = \dfrac{\sin\theta + \sin^2\theta}{(\cos\theta)(1 + \sin\theta)}$

b) $\dfrac{\sin^2\theta}{1 - \cos\theta} = 1 + \cos\theta$

$$= \frac{\sin\theta\,\cancel{(1+\sin\theta)}}{\cos\theta\,\cancel{(1+\sin\theta)}}$$

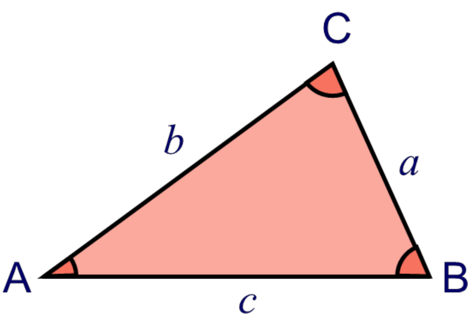
$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta = LS$$

where $\cos\theta \neq 0$
$\sin\theta \neq -1$

The Sine Law

Use this when we know an angle-side pair and one other side or angle.



Solving for a side length use:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Sub in the values and solve to find the side length.

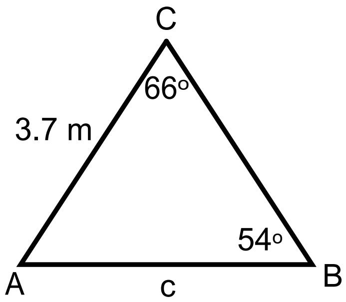
Solving for an angle use:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Sub in the values and find the inverse sine of the ratio.

Example

Find the side length c



$$\frac{c}{sin(C)} = \frac{b}{sin(B)}$$

$$\frac{c}{sin(66)} = \frac{3.7}{sin(54)}$$

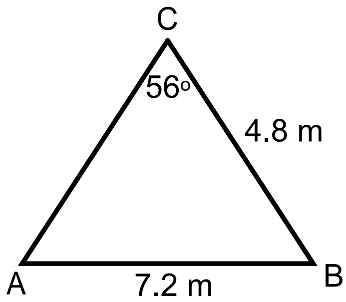
$$c = \frac{3.7sin(66)}{sin(54)}$$

c = 4.178

c is about 4.2 metres long

Example

Find the size of angle A



$$\frac{sin(A)}{a} = \frac{sin(C)}{c}$$

$$\frac{sin(A)}{4.8} = \frac{sin(56)}{7.2}$$

$$sin(A) = \frac{4.8sin(56)}{7.2}$$

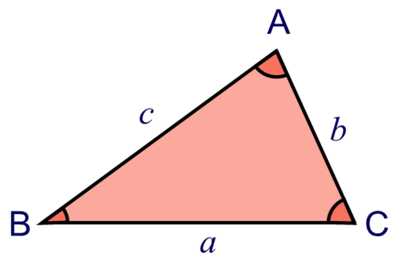
$$A = sin^{-1}(0.5527)$$

A = 33.55

Angle A is about 34°

The Cosine Law

Use this when you know all three sides of a triangle OR you know two sides and the contained angle (SAS).



Solving for a side length use:

$a^2 = b^2 + c^2 - 2bc\cos(A)$

$b^2 = a^2 + c^2 - 2ac\cos(B)$

$c^2 = a^2 + b^2 - 2ab\cos(C)$

Sub in the values and find the square root of the answer.

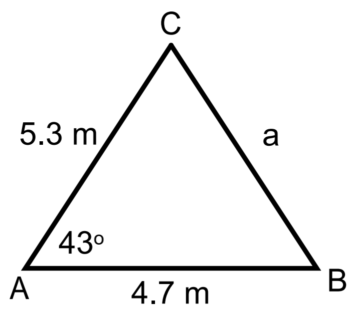
Solving for an angle use:

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc} \quad \cos(B) = \frac{a^2 + c^2 - b^2}{2ac} \quad \cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

Sub in the values and find the inverse cosine of the ratio.

Example

Solve to find side length a



$a^2 = b^2 + c^2 - 2bc\cos(A)$

$a^2 = 5.3^2 + 4.7^2 - 2(5.3)(4.7)\cos(43)$

$a^2 = 13.74395859$

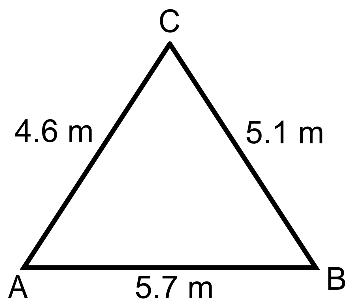
$a = \sqrt{ANS}$

a = 3.707

a is about 3.7 metres long

Example

Find the size of angle B



$$cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$$

$$cos(B) = \frac{5.1^2 + 5.7^2 - 4.6^2}{2(5.1)(5.7)}$$

$$cos(B) = \frac{37.34}{58.14}$$

$$B = cos^{-1}(0.6422)$$

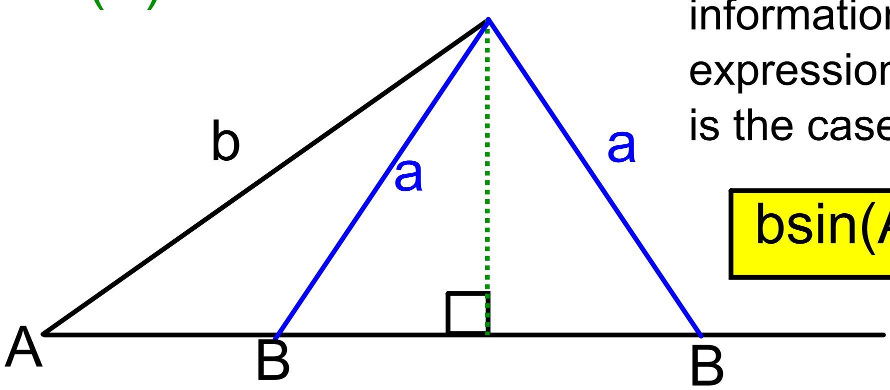
B = 50.04

Angle B is about 50°

The Ambiguous Case

$$\sin(A) = \frac{a}{b}$$

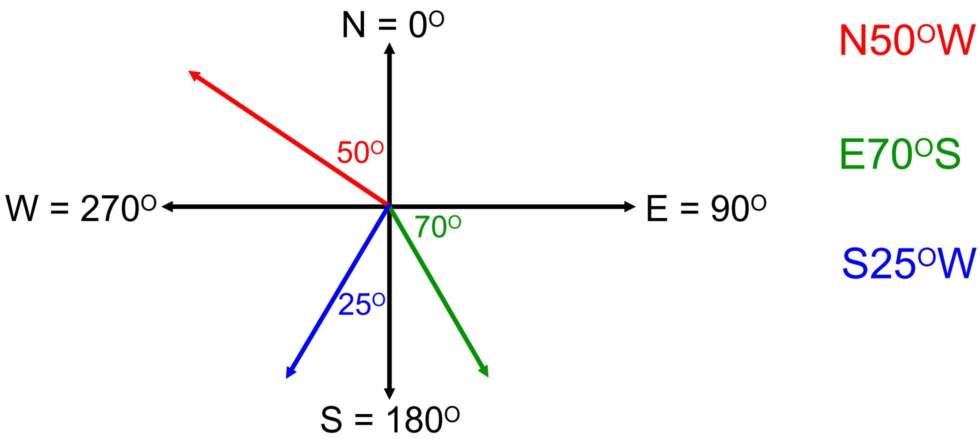
$\text{bsin}(A) = a$



This is when there could be TWO possible triangles based upon the given information. Use the expression to check if this is the case.

$\text{bsin}(A) < a < b$

Compass Direction

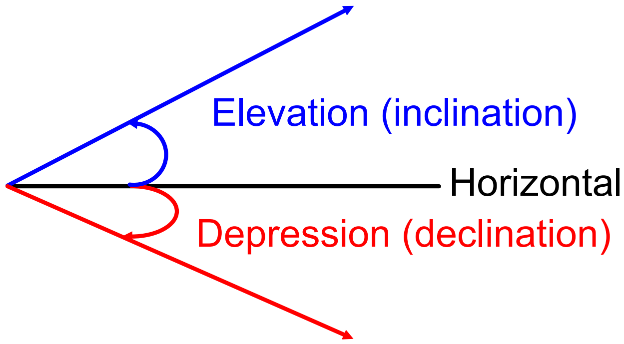


You may also use **BEARINGS** to measure direction. These are always:

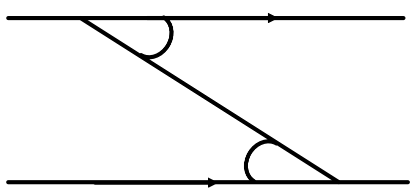
- measured from North
- measured in a clockwise direction
- given with 3 digits

Other Word Problems

Recall angles of elevation and depression. These are always measured with reference to the HORIZONTAL.

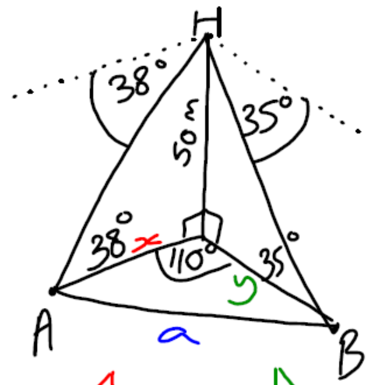


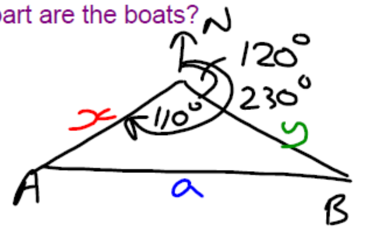
You can also make use of the property of alternate (Z) angles being equal.

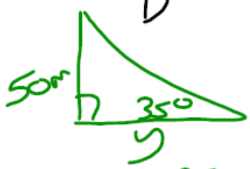


Also if you know two angles in a triangle you can use the 180 rule to find the third.

Example:
Bunter is on a 50m high bridge and sees two boats anchored below. From his position, boat A has a bearing of 230°, and boat B has a bearing of 120°. Bunter estimates the angles of depressions to be 38° for boat A and 35° for boat B. How far apart are the boats?







$\tan 38 = \frac{50}{x}$

$x = \frac{50}{\tan 38}$

$x = 64.0\text{m}$

$\tan 35 = \frac{50}{y}$

$y = \frac{50}{\tan 35}$

$y = 71.4\text{m}$

$a^2 = x^2 + y^2 - 2xy\cos A$

$a^2 = 64.0^2 + 71.4^2 - 2(64.0)(71.4)\cos 110$

$a^2 = 12319.75$

$a = \sqrt{\text{ANS}}$

$a = 111.0\text{m}$