Review

1. Topics:

- Primary Trig Ratios (SOH CAH TOA)
- Reciprocal Trig Ratios
- Special Angles
- Angle Terminology (Principle, Negative, RAA etc.)
- Unit Circle
- Solving Trig Equations for angles between 0 and 360
- Trig Identities
- Sine Law and Cosine Law
- The Ambiguous Case
- 3D Trig Problems

2. Review Questions

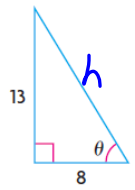
Nelson Page 338 #s 1 – 4, 7acd, 9, 11 & 12

Solutions

1. i) For each triangle, state the reciprocal trigonometric ratios for angle θ.
ii) Calculate the value of θ to the nearest degree.

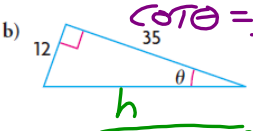
$\csc\theta = \frac{hyp}{opp}$ $\sec\theta = \frac{hyp}{adj}$ $\cot\theta = \frac{adj}{opp}$

a)



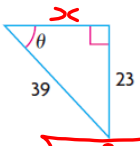
$h = \sqrt{8^2+13^2}$
$h = \sqrt{64+169}$
$h = \sqrt{233}$
$\csc\theta = \frac{\sqrt{233}}{13}$
$\sec\theta = \frac{\sqrt{233}}{8}$
$\cot\theta = \frac{8}{13}$
$\theta = \tan^{-1}\left(\frac{13}{8}\right)$
$\theta = 58°$

b)



$h = \sqrt{12^2+35^2}$
$h = \sqrt{144+1225}$
$h = \sqrt{1369}$
$h = 37$
$\csc\theta = \frac{37}{12}$
$\sec\theta = \frac{37}{35}$
$\cot\theta = \frac{35}{12}$
$\theta = \tan^{-1}\left(\frac{12}{35}\right)$
$\theta = 19°$

c)



$x = \sqrt{39^2-23^2}$
$x = \sqrt{1521-529}$
$x = \sqrt{992}$
$x = \sqrt{16\times 62}$
$x = 4\sqrt{62}$
$\csc\theta = \frac{39}{23}$
$\sec\theta = \frac{39}{4\sqrt{62}}$
$\cot\theta = \frac{4\sqrt{62}}{23}$
$\theta = \sin^{-1}\left(\frac{23}{39}\right)$
$\theta = 36°$

2. Determine the exact value of each trigonometric expression. Express your answers in simplified radical form.
a) $(\sin 45°)(\cos 45°) + (\sin 30°)(\cos 60°)$
b) $(1 - \tan 45°)(\sin 30°)(\cos 30°)(\tan 60°)$
c) $\tan 30° + 2(\sin 45°)(\cos 60°)$

a) $\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$
$= \frac{2}{4} + \frac{1}{4}$
$= \frac{3}{4}$

b) $(1-1)\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\left(\sqrt{3}\right)$
$= (0)\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\left(\sqrt{3}\right)$
$= 0$

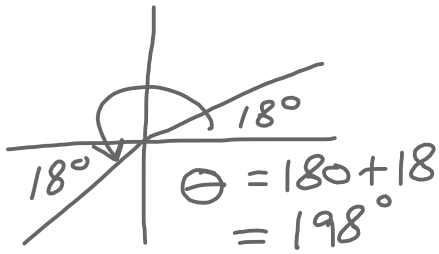
c) $\frac{1}{\sqrt{3}} + 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$
$= \frac{\sqrt{3}}{3} + \frac{\sqrt{2}}{2}$
$= \frac{2\sqrt{3}+3\sqrt{2}}{6}$

3. i) State the sign of each trigonometric ratio. Use a calculator to determine the value of each ratio.

a) $\tan 18°$ b) $\sin 205°$ c) $\cos(-55°)$

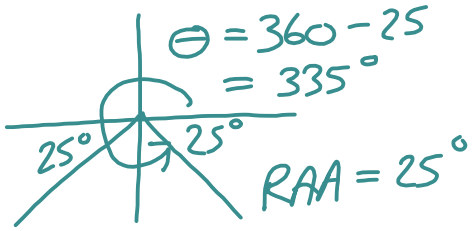
ii) For each trigonometric ratio, determine the principal angle and, where appropriate, the related acute angle. Then sketch another angle that has the equivalent ratio. Label the principal angle and the related acute angle on your sketch.

a) $18° \rightarrow$ Q1
$\Rightarrow$ POSITIVE
TAN 18 = 0.3249



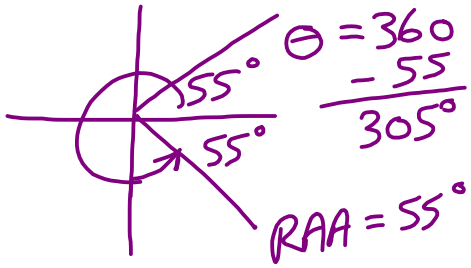
$\theta = 180 + 18$
$= 198°$

b) $205° \rightarrow$ Q3
$\Rightarrow$ NEGATIVE
SIN 205 = −0.4226



$\theta = 360 - 25$
$= 335°$
RAA = 25°

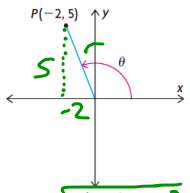
c) $-55° \rightarrow$ Q4
$\Rightarrow$ POSITIVE
COS(−55) = 0.5736



$\theta = 360$
$- 55$
$305°$
RAA = 55°

4. For each sketch, state the primary trigonometric ratios associated with angle θ. Express your answers in simplified radical form.

a)



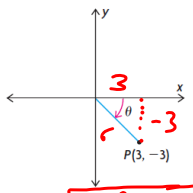
$r = \sqrt{(-2)^2 + 5^2}$
$r = \sqrt{4 + 25}$
$r = \sqrt{29}$

$\sin\theta = \frac{5}{\sqrt{29}}$
$= \frac{5\sqrt{29}}{29}$

$\cos\theta = \frac{-2}{\sqrt{29}}$
$= \frac{-2\sqrt{29}}{29}$

$\tan\theta = -\frac{5}{2}$

b)



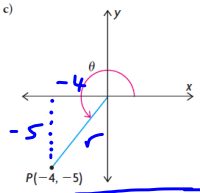
$r = \sqrt{(3)^2 + (-3)^2}$
$r = \sqrt{9 + 9}$
$r = \sqrt{18}$

$\sin\theta = \frac{-3}{\sqrt{18}}$
$= \frac{-3\sqrt{18}}{18}$
$= \frac{-3(3)\sqrt{2}}{18}$
$= -\frac{\sqrt{2}}{2}$

$\cos\theta = \frac{3}{\sqrt{18}}$
$= \frac{\sqrt{2}}{2}$

$\tan\theta = \frac{-3}{3}$
$= -1$

c)



$r = \sqrt{(-4)^2 + (-5)^2}$
$r = \sqrt{16 + 25}$
$r = \sqrt{41}$

$\sin\theta = \frac{-5}{\sqrt{41}}$
$= \frac{-5\sqrt{41}}{41}$

$\cos\theta = \frac{-4}{\sqrt{41}}$
$= \frac{-4\sqrt{41}}{41}$

$\tan\theta = \frac{-5}{-4}$
$= \frac{5}{4}$

7. Prove each identity. State any restrictions on the variables if all angles vary from 0° to 360°.

a) $\tan\alpha\cos\alpha = \sin\alpha$

$\frac{\sin\alpha}{\cos\alpha} \times \frac{\cos\alpha}{1}$

$\sin\alpha = RS$

c) $1 - \cos^2 x = \frac{\sin x \cos x}{\cot x}$

$\sin x \cos x \div \frac{\cos x}{\sin x}$

$\frac{\sin x}{1} \times \frac{\cos x}{1} \times \frac{\sin x}{\cos x}$

$\sin^2 x$

$1 - \cos^2 x = LS$

$\cot x \neq 0$

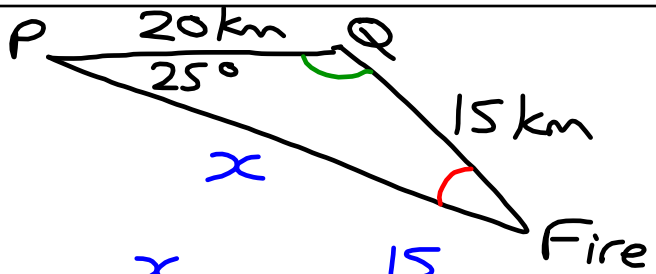
d) $\sec\theta\cos\theta + \sec\theta\sin\theta = 1 + \tan\theta$

$\frac{1}{\cos\theta} \times \cos\theta + \frac{1}{\cos\theta} \times \sin\theta$

$\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$1 + \tan\theta = RS$

9. Two forest fire stations, P and Q, are 20.0 km apart. A ranger at station Q sees a fire 15.0 km away. If the angle between the line PQ and the line from P to the fire is 25°, how far, to the nearest tenth of a kilometre, is station P from the fire?



$\frac{\sin F}{20} = \frac{\sin 25}{15}$

$\sin F = \frac{20\sin 25}{15}$

$\angle F = \sin^{-1}(\text{ANS})$

$= 34.3°$

$\angle Q = 180 - 25 - 34.3$

$= 120.7°$

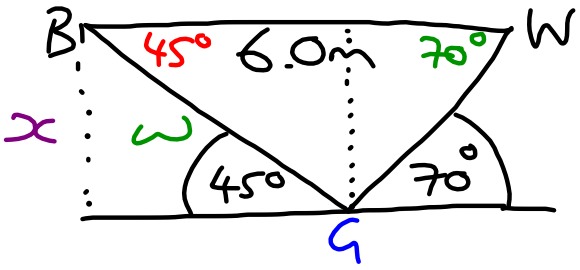
$\frac{x}{\sin Q} = \frac{15}{\sin 25}$

$\frac{x}{\sin 120.7} = \frac{15}{\sin 25}$

$x = \frac{15\sin 120.7}{\sin 25}$

$x = 30.5\text{ km}$

11. Two spotlights, one blue and the other white, are placed 6.0 m apart on a track on the ceiling of a ballroom. A stationary observer standing on the ballroom floor notices that the angle of elevation is 45° to the blue spotlight and 70° to the white one. How high, to the nearest tenth of a metre, is the ceiling of the ballroom?



$\angle G = 180 - 45 - 70$

$= 65°$

Z angle $\Rightarrow \angle B = 45°$

Z angle $\Rightarrow \angle W = 70°$

$\frac{w}{\sin 70} = \frac{6.0}{\sin 65}$

$w = \frac{6.0 \sin 70}{\sin 65}$

$w = 6.22\text{m}$

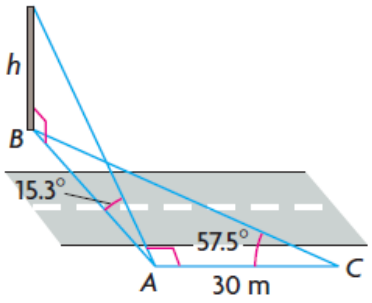
$\sin 45 = \frac{x}{w}$

$\sin 45 = \frac{x}{6.22}$

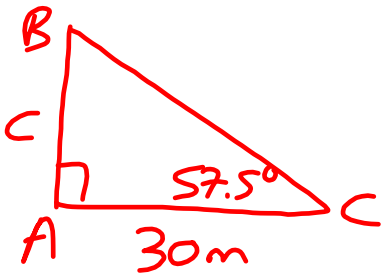
$6.22 \sin 45 = x$

$x = 4.4\text{m}$

12. To determine the height of a pole across a road, Justin takes two measurements. He stands at point A directly across from the base of the pole and determines that the angle of elevation to the top of the pole is 15.3°. He then walks 30 m parallel to the freeway to point C, where he sees that the base of the pole and point A are 57.5° apart. From point A, the base of the pole and point C are 90.0° apart. Calculate the height of the pole to the nearest metre.



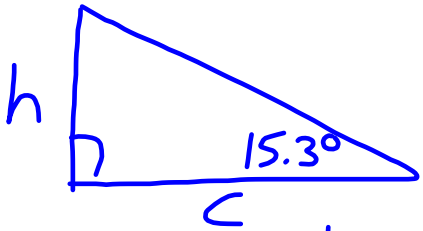
Ground level △



$\tan 57.5° = \frac{c}{30}$

$30 \tan 57.5 = c$

$c = 47.1\text{m}$



$\tan 15.3 = \frac{h}{c}$

$\tan 15.3 = \frac{h}{47.1}$

$47.1 \tan 15.3 = h$

$h = 12.9 \Rightarrow 13 \text{ metres}$