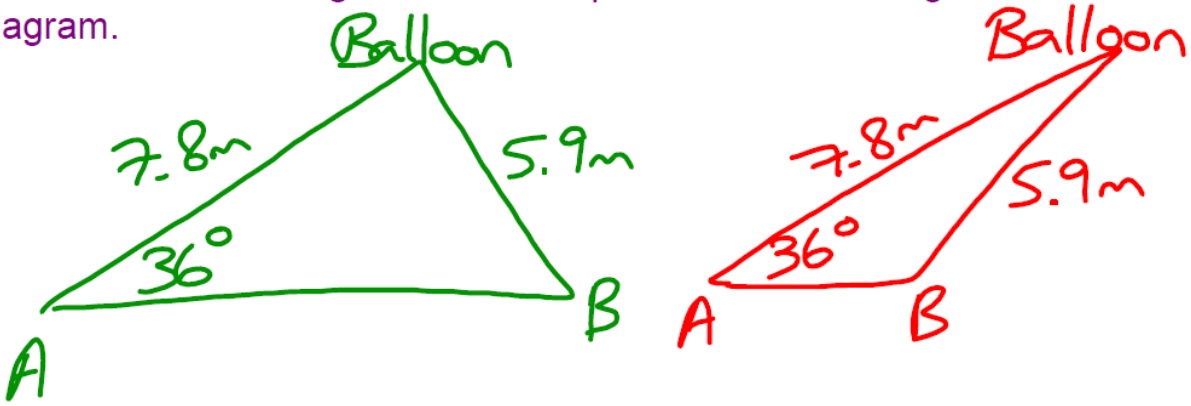
Warm Up

Albert and Belle are scientists testing weather balloons. They are both holding the balloon by a rope, getting ready to launch the balloon. Albert's rope holding the balloon is making a 36° angle with the ground and is 7.8 metres long and Belle's rope is 5.9 metres long. Draw a diagram.



Sine Law and the Ambiguous Case

Lesson objectives

- I know how to determine when the ambiguous case occurs
- I know how to determine both solutions to the ambiguous case

1.1

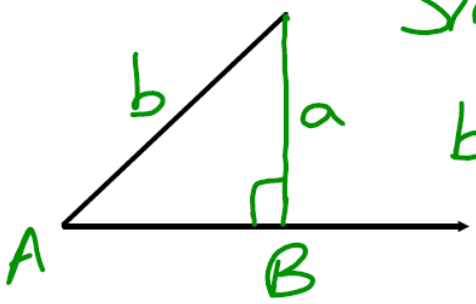
Lesson objectives | Teachers' notes | Lesson notes

Nelson Page 318 #s 2, 3 & 5

Recap Sine and Cosine Law

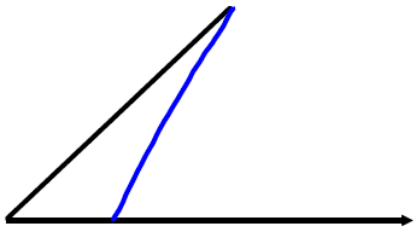
Given Info:	To Be Found:	Law Required:
1. Two angles and any side (AAS)	side	sine law
2. Two sides and a contained angle (SAS)	side	cosine law
3. Three sides (SSS)	angle	cosine law
4. 2 sides and an angle opposite one of them (SSA)	angle	sine law

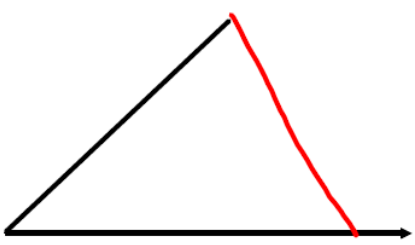
Draw a line to complete this shape and make it a triangle.



$\sin A = \frac{a}{b}$

$b \sin A = a$

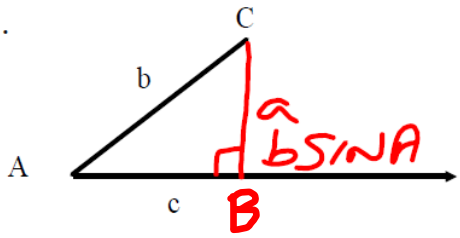




Ambiguous Case Options:

Given angle A < 90 degrees (acute)

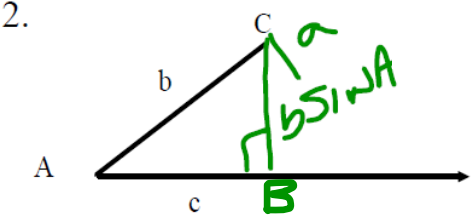
1.



$a = b\sin(A)$

ONE TRIANGLE

2.



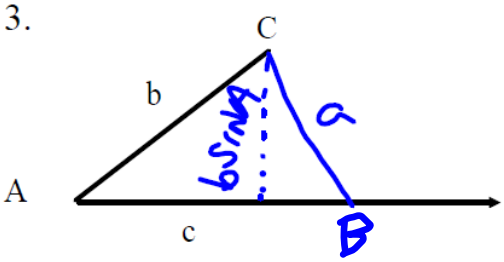
$a < b\sin(A)$

NO TRIANGLE

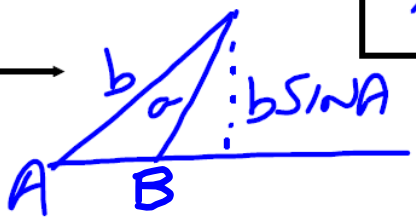
Ambiguous Case Options:

Given angle A < 90 degrees (acute)

3.

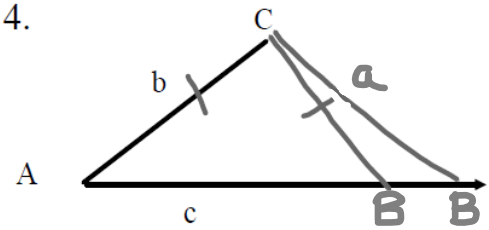


$b\sin(A) < a < b$



TWO TRIANGLES

4.



$a \geq b$

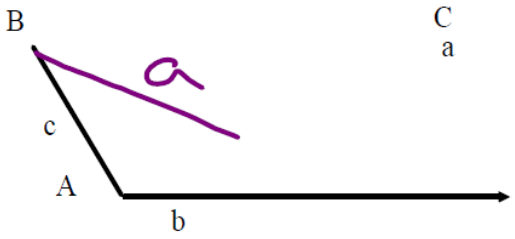
ONE TRIANGLE

a = b or a ≥ b

Ambiguous Case Options:

Given angle A > 90 degrees (obtuse)

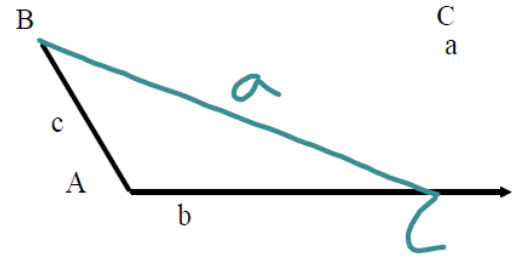
1.



$a \leq b$

NO TRIANGLE

2.



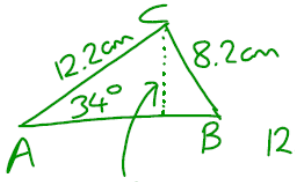
$a > b$

ONE TRIANGLE

Determine the number of solutions, then find the solutions and solve the obtuse triangle

Angle A = 34°, a = 8.2cm, b = 12.2cm

1. Draw the picture (remember side-side-angle)
2. Determine which case you have
3. Find the angle or angle(s) possible.



$bSINA < a < b$

$12.2SIN34 < 8.2 < 12.2$

$6.82 < 8.2 < 12.2$

$\Rightarrow$ TRUE $\Rightarrow$ 2 POSSIBLE TRIANGLES

$\frac{SIN34}{8.2} = \frac{SINB}{12.2}$

$\frac{12.2\,SIN34}{8.2} = SINB$

$SIN^{-1}(ANS) = B$

$\angle B = 56.3°$

$\Rightarrow \angle C = 180 - 34 - 56.3$

$= 89.7°$

$\frac{c}{SIN89.7} = \frac{8.2}{SIN34}$

$c = \frac{8.2\,SIN89.7}{SIN34}$

$c = 14.7cm$

Using CAST

$\angle B = 180 - 56.3$

$= 123.7°$

$\Rightarrow \angle C = 180 - 34 - 123.7$

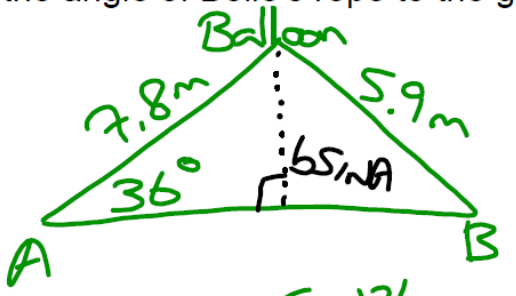
$= 22.3°$

$\frac{c}{SIN22.3} = \frac{8.2}{SIN34}$

$c = \frac{8.2\,SIN22.3}{SIN34}$

$c = 5.6cm$

Look at the situation with Albert and Belle from the warm up. Determine the angle of Belle's rope to the ground.

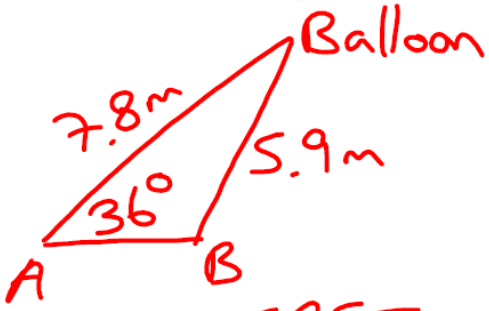


$$\frac{\sin B}{7.8} = \frac{\sin 36}{5.9}$$

$$\sin B = \frac{7.8\sin 36}{5.9}$$

$$\angle B = \sin^{-1}(\text{ANS})$$

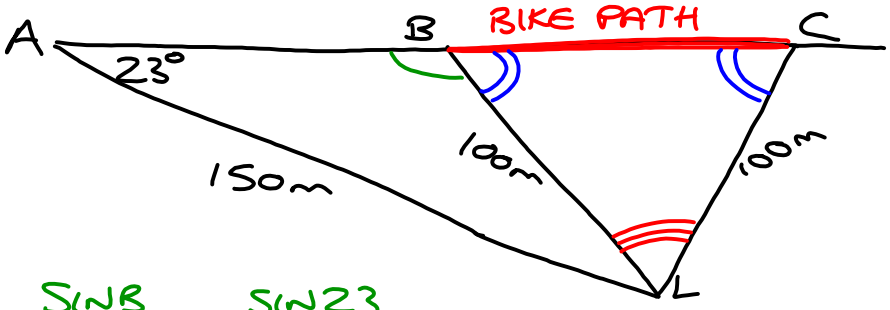
$$\angle B = 51^\circ$$



Using CAST

$$\angle B = 180 - 51$$
$$= 129^\circ$$

A light in a park can illuminate effectively up to a distance of 100m. A point on a bike path is 150m from the light. The sight line to the light makes an angle of 23° with the bike path. What length of the bike path, to the nearest metre, is effectively illuminated?



$$\frac{\sin B}{150} = \frac{\sin 23}{100}$$

$$\sin B = \frac{150\sin 23}{100}$$

$$\angle B = \sin^{-1}(\text{ANS})$$

$$\angle B = 35.9^\circ$$

But $\angle B$ could be obtuse

$$\Rightarrow \angle B = 180 - 35.9$$
$$= 144.1^\circ$$

$$\angle B = \angle C$$
$$= 180 - 144.1$$
$$= 35.9^\circ$$

$$\angle L = 180 - 35.9 - 35.9$$
$$\angle L = 108.2^\circ$$

$$BC^2 = 100^2 + 100^2 - 2(100)(100)\cos(108.2)$$
$$BC^2 = 26246.69837$$
$$\Rightarrow BC = 162.0\text{m}$$