Warm Up:

Prove the following identity:

$\cos\theta(1+\sec\theta)(\cos\theta-1) = -\sin^2\theta$

$\cos\theta\left(1+\frac{1}{\cos\theta}\right)(\cos\theta-1)$

$\left(\cos\theta+\frac{\cos\theta}{\cos\theta}\right)(\cos\theta-1)$

$(\cos\theta+1)(\cos\theta-1)$

$\cos^2\theta-1$

$=-(-\cos^2\theta+1)$

$=-(1-\cos^2\theta)$

$=-\sin^2\theta = RS$

Recall

$\sin^2\theta+\cos^2\theta=1$

Sine and Cosine Law

Lesson objectives

- I know how to use the Sine Law
- I know how to use the Cosine Law

1.1

Lesson objectives | Teachers' notes | Lesson notes

Nelson Page 318 #s 1, 7 & 12 and Page 325 #s 1, 2 & 10

Sine Law

Recall:
- The sine law is used when we don't have a right triangle

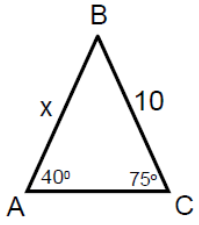
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{OR} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- To use the sine law we must have at least one pair (side and angle) and have either the side or angle to match what we are looking for.

- We use the version with sine in the numerator if we are solving for an angle and we use the side length in the numerator if we are solving for a length.

Example:
Determine the length of the unknown side or measure of the unknown angle.

a)

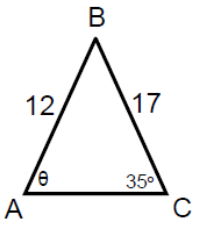


$\frac{x}{\sin 75} = \frac{10}{\sin 40}$

$x = \frac{10 \sin 75}{\sin 40}$

$x = 15$ units

b)



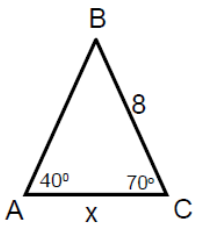
$\frac{\sin\theta}{17} = \frac{\sin 35}{12}$

$\sin\theta = \frac{17 \sin 35}{12}$

$\theta = \sin^{-1}(\text{ANS})$

$\theta = 54°$

c)



$\angle B = 180 - 40 - 70$

$70°$

$\frac{x}{\sin 70} = \frac{8}{\sin 40}$

$x = \frac{8 \sin 70}{\sin 40}$

$x = 11.7$ units

The Cosine Law

Recall:
- The cosine law is also used when we aren't given a right triangle

$c^2 = a^2 + b^2 - 2abcosC$ $a^2 = b^2 + c^2 - 2bccosA$

$b^2 = a^2 + c^2 - 2accosB$ $\cos A = \frac{b^2+c^2-a^2}{2bc}$

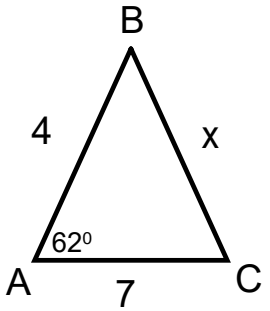
- To use the cosine law you must have:
  1. All 3 sides OR
  2. 2 sides and the contained angle

In grade 10 you dealt, solely with acute triangles, this year we extend the cosine law to obtuse triangles.

Example:

Determine the length of the unknown side or measure of the unknown angle.

a)



$a^2 = b^2 + c^2 - 2bc\cos A$

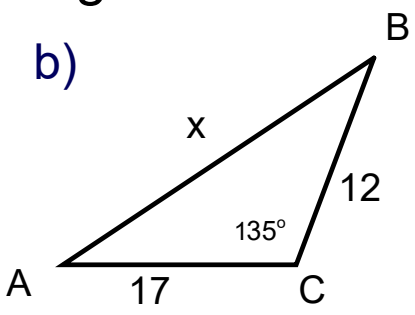
$x^2 = 7^2 + 4^2 - 2(7)(4)\cos 62$

$x^2 = 38.70959248$

$x = \sqrt{ANS}$

$x = 6.2$ units

b)



$c^2 = a^2 + b^2 - 2ab\cos C$

$x^2 = 12^2 + 17^2 - 2(12)(17)\cos(135)$

$x^2 = 721.4995667$

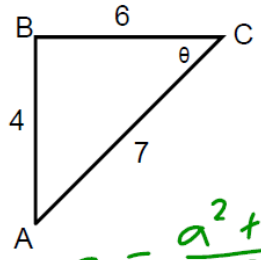
$x = \sqrt{ANS}$

$x = 26.9$ units

Example:

Determine the length of the unknown side or measure of the unknown angle.

c)



$\cos C = \frac{a^2+b^2-c^2}{2ab}$

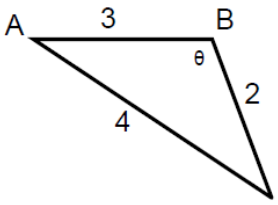
$\cos\theta = \frac{6^2+7^2-4^2}{2(6)(7)}$

$\cos\theta = \frac{69}{84}$

$\theta = \cos^{-1}(\text{ANS})$

$\theta = 35°$

d)



$\cos B = \frac{a^2+c^2-b^2}{2ac}$

$\cos\theta = \frac{2^2+3^2-4^2}{2(2)(3)}$

$\cos\theta = \frac{-3}{12}$

$\theta = \cos^{-1}(\text{ANS})$

$\theta = 104°$