Solutions

2. Simplify each expression.

a) $(1 - \sin\alpha)(1 + \sin\alpha)$

$= 1 + \cancel{\sin\alpha} - \cancel{\sin\alpha} - \sin^2\alpha$

$= 1 - \sin^2\alpha$

Pythagorean Identity

$\sin^2\alpha + \cos^2\alpha = 1$

$\Rightarrow \cos^2\alpha$

c) $\cos^2\alpha + \sin^2\alpha$

$= 1 - \cancel{\sin^2\alpha} + \cancel{\sin^2\alpha}$

$= 1$

b) $\dfrac{\tan\alpha}{\sin\alpha}$

$= \dfrac{\sin\alpha}{\cos\alpha} \div \sin\alpha$

$= \dfrac{\cancel{\sin\alpha}}{\cos\alpha} \times \dfrac{1}{\cancel{\sin\alpha}}$

$= \dfrac{1}{\cos\alpha}$

$= \sec\alpha$

d) $\cot\alpha \sin\alpha$

$= \dfrac{\cos\alpha}{\cancel{\sin\alpha}} \times \dfrac{\cancel{\sin\alpha}}{1}$

$= \cos\alpha$

3. Factor each expression.

a) $1 - \cos^2 \theta$

Difference of squares

$\Rightarrow (1+\cos\theta)(1-\cos\theta)$

c) $\sin^2 \theta - 2 \sin \theta + 1$

$= (\sin\theta - 1)(\sin\theta - 1)$

$= (\sin\theta - 1)^2$

If it helps think of it as $x^2 - 2x + 1$ where $x = \sin\theta$

b) $\sin^2 \theta - \cos^2 \theta$

Difference of squares

$\Rightarrow (\sin\theta + \cos\theta)(\sin\theta - \cos\theta)$

d) $\cos \theta - \cos^2 \theta$

$= \cos\theta(1 - \cos\theta)$

5. Prove each identity. State any restrictions on the variables.

a) $\dfrac{\sin x}{\tan x} = \cos x$

c) $\dfrac{1}{\cos \alpha} + \tan \alpha = \dfrac{1 + \sin \alpha}{\cos \alpha}$

Look to start with the more complicated side or the one with TAN or reciprocal ratios

$\sin x \div \tan x$

$= \sin x \div \dfrac{\sin x}{\cos x}$

$= \dfrac{\cancel{\sin x}}{1} \times \dfrac{\cos x}{\cancel{\sin x}}$

$= \cos x = RS$

where $\tan x \neq 0$

$\dfrac{1}{\cos\alpha} + \dfrac{\sin\alpha}{\cos\alpha}$

$= \dfrac{1+\sin\alpha}{\cos\alpha} = RS$

where $\cos\alpha \neq 0$

5. Prove each identity. State any restrictions on the variables.

b) $\dfrac{\tan\theta}{\cos\theta} = \dfrac{\sin\theta}{1-\sin^2\theta}$

$\dfrac{\sin\theta}{\cos\theta} \div \cos\theta$

$= \dfrac{\sin\theta}{\cos\theta} \times \dfrac{1}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos^2\theta}$

$= \dfrac{\sin\theta}{1-\sin^2\theta} = RS$

where $\cos\theta \neq 0$

d) $1 - \cos^2\theta = \sin\theta\cos\theta\tan\theta$

$\sin\theta \times \cancel{\cos\theta} \times \dfrac{\sin\theta}{\cancel{\cos\theta}}$

$= \sin^2\theta$

$= 1-\cos^2\theta = LS$

where $\tan\theta \neq 0$

7. Simplify each trigonometric expression.

a) $\sin\theta\cot\theta - \sin\theta\cos\theta$

$= \dfrac{\cancel{\sin\theta}}{1} \times \dfrac{\cos\theta}{\cancel{\sin\theta}} - \sin\theta\cos\theta$

$= \cos\theta - \sin\theta\cos\theta$

$= \cos\theta(1-\sin\theta)$

b) $\cos\theta(1 + \sec\theta)(\cos\theta - 1)$

$= \cos\theta\left(1+\dfrac{1}{\cos\theta}\right)(\cos\theta-1)$

$= (\cos\theta+1)(\cos\theta-1)$

$= \cos^2\theta \cancel{-\cos\theta} \cancel{+\cos\theta} - 1$

$= \cos^2\theta - 1$

$= -(-\cos^2\theta+1)$

$= -(1-\cos^2\theta)$

$= -\sin^2\theta$

7. Simplify each trigonometric expression.

c) $(\sin x + \cos x)(\sin x - \cos x) + 2\cos^2 x$

$= \sin^2 x - \cos^2 x + 2\cos^2 x$

$= \sin^2 x + \cos^2 x$

$= 1$

d) $\dfrac{\csc^2\theta - 3\csc\theta + 2}{\csc^2\theta - 1}$

$= \dfrac{(\csc\theta - 2)\cancel{(\csc\theta - 1)}}{(\csc\theta + 1)\cancel{(\csc\theta - 1)}}$

$= \dfrac{\csc\theta - 2}{\csc\theta + 1}$

where $\csc\theta \neq -1$

8. Prove each identity. State any restrictions on the variables.

a) $\dfrac{\sin^2\phi}{1 - \cos\phi} = 1 + \cos\phi$

$= \dfrac{1 - \cos^2\phi}{1 - \cos\phi}$

$= \dfrac{(1 + \cos\phi)\cancel{(1 - \cos\phi)}}{\cancel{1 - \cos\phi}}$

$= 1 + \cos\phi = RS$

where $\cos\phi \neq 1$

b) $\dfrac{\tan^2\alpha}{1 + \tan^2\alpha} = \sin^2\alpha$

$= \dfrac{\sin^2\alpha}{\cos^2\alpha} \div 1 + \tan^2\alpha$

$= \dfrac{\sin^2\alpha}{\cos^2\alpha} \times \dfrac{1}{1 + \frac{\sin^2\alpha}{\cos^2\alpha}}$

$= \dfrac{\sin^2\alpha}{\cos^2\alpha} \times \dfrac{1}{\frac{\cos^2\alpha}{\cos^2\alpha} + \frac{\sin^2\alpha}{\cos^2\alpha}}$

$= \dfrac{\sin^2\alpha}{\cos^2\alpha} \times \dfrac{1}{\frac{\cos^2\alpha + \sin^2\alpha}{\cos^2\alpha}}$

$= \dfrac{\sin^2\alpha}{\cos^2\alpha} \times \dfrac{1}{\frac{1}{\cos^2\alpha}}$

$= \dfrac{\sin^2\alpha}{\cancel{\cos^2\alpha}} \times \dfrac{\cancel{\cos^2\alpha}}{1}$

where $\tan\alpha \neq -1$

$= \sin^2\alpha = RS$

8. Prove each identity. State any restrictions on the variables.

c) $\cos^2 x = (1 - \sin x)(1 + \sin x)$

$= 1 \cancel{+ \sin x} \cancel{- \sin x} - \sin^2 x$

$= 1 - \sin^2 x$

$= \cos^2 x = LS$

d) $\sin^2 \theta + 2\cos^2 \theta - 1 = \cos^2 \theta$

$= (1 - \cos^2 \theta) + 2\cos^2 \theta - 1$

$= \cancel{1} - \cos^2 \theta + 2\cos^2 \theta \cancel{- 1}$

$= \cos^2 \theta = RS$