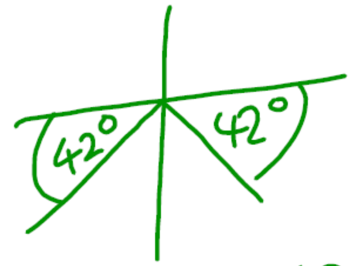
Warm Up:

Find all solutions for θ, where $0 \leq \theta \leq 360$.

a) $\sin\theta = -0.6723$ b) $\cos\theta = 0.4291$ c) $\tan\theta = -1.237$

$\theta = \sin^{-1}(-0.6723)$

$\theta = -42^\circ$



$\theta = 180 + 42$
$= 222^\circ$

$\theta = 360 - 42$
$= 318^\circ$

$\theta = \cos^{-1}(0.4291)$

$\theta = 65^\circ$

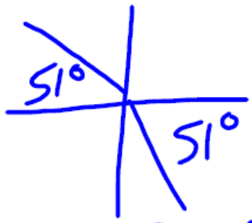
$\theta = 65^\circ$

$\theta = 360 - 65$

$\theta = 295^\circ$

$\theta = \tan^{-1}(-1.237)$

$\theta = -51^\circ$



$\theta = 180 - 51$
$= 129^\circ$

$\theta = 360 - 51$
$= 309^\circ$

Trigonometric Identities

Lesson objectives

- I understand the Quotient Identity
- I understand the Pythagorean Identity
- I know how to use the two basic identities to simplify trigonometric expressions
- I know the different strategies used to prove trigonometric identities
- I understand that the identities need to be split into left side / right side
- I know how to apply the basic identities to make both sides equal

1.1 Lesson objectives | Teachers' notes | Lesson notes

Nelson Page 310 #s 2, 3, 5, 7 & 8abcd

Trigonometric Identities

<u>Identity</u>: a mathematical statement that is true for all values of the given variable. If the identity involves fractions, the denominators cannot be zero. Any restrictions on a variable must be stated.

- Identities are like puzzles: we have to rearrange equations until they match
- We set up identities in the form <u>LS | RS</u> (left side, right side) and cannot move terms or factors from one side to the other.
- We have 2 main identities from which many others can be formed.

1. Quotient Identity (Q.I.):

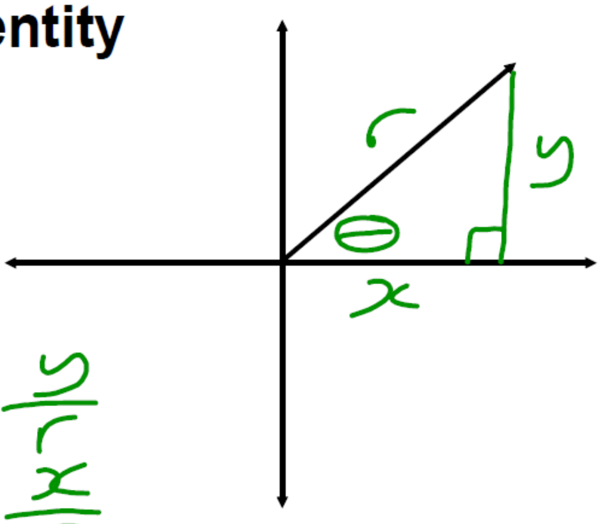
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

2. Pythagorean Identity (P.I.):

$$\sin^2\theta + \cos^2\theta = 1$$

Proof of the Quotient Identity

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$



L.S. $= \tan\theta$

$= \dfrac{y}{x}$

R.S. $= \dfrac{\sin\theta}{\cos\theta}$

$\sin\theta = \dfrac{y}{r}$

$\cos\theta = \dfrac{x}{r}$

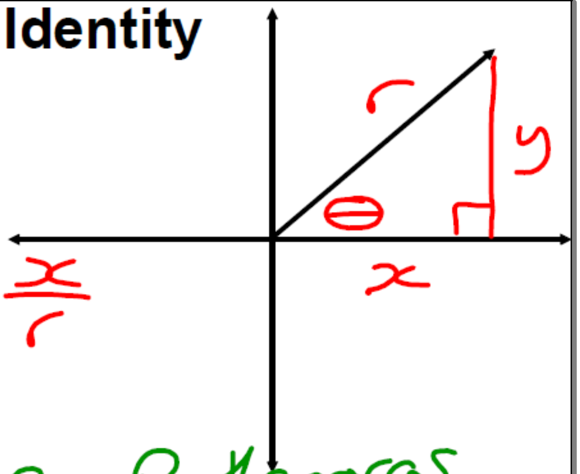
$\Rightarrow \dfrac{y}{r} \div \dfrac{x}{r}$

$\dfrac{y}{\cancel{r}} \times \dfrac{\cancel{r}}{x} = \dfrac{y}{x} = LS$

Proof of the Pythagorean Identity

$\sin^2\theta + \cos^2\theta = 1$

L.S. = $\sin^2\theta + \cos^2\theta$ R.S. = 1



$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$

$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$

By Pythagoras

$x^2 + y^2 = r^2$

$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$

$= \frac{x^2+y^2}{r^2} \Rightarrow \frac{r^2}{r^2} = 1 = RS$

Reciprocal Identities

Recall: we also have our reciprocal identities:

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Simplify the following expressions:

a) $\cos\theta\tan\theta$

$= \frac{\cancel{\cos\theta}}{1} \times \frac{\sin\theta}{\cancel{\cos\theta}}$

$= \sin\theta$

b) $\cot\theta\sin\theta$

$= \frac{\cos\theta}{\cancel{\sin\theta}} \times \frac{\cancel{\sin\theta}}{1}$

$= \cos\theta$

Pythagorean Identity

We can rearrange the Pythagorean Identity to get two slightly different identities:

$\sin^2\theta + \cos^2\theta = 1$

1. $\cancel{\sin^2\theta} + \cos^2\theta - \cancel{\sin^2\theta} = 1 - \sin^2\theta$ (subtract $\sin^2\theta$ from both sides)

$\cos^2\theta = 1 - \sin^2\theta$

2. $\sin^2\theta + \cancel{\cos^2\theta} - \cancel{\cos^2\theta} = 1 - \cos^2\theta$ (subtract $\cos^2\theta$ from both sides)

$\sin^2\theta = 1 - \cos^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} \qquad \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta \qquad \tan^2\theta + 1 = \sec^2\theta$$

Simplify the following expressions:

a) $\frac{\sin\theta}{\csc\theta} + \frac{\cos\theta}{\sec\theta}$

$= \frac{\sin\theta}{1} \times \frac{1}{\csc\theta} + \frac{\cos\theta}{1} \times \frac{1}{\sec\theta}$

$= (\sin\theta)(\sin\theta) + (\cos\theta)(\cos\theta)$

$= \sin^2\theta + \cos^2\theta$

$= 1$

b) $\sin\theta(\csc\theta - \sin\theta)$

$\frac{\cancel{\sin\theta}}{1} \times \frac{1}{\cancel{\sin\theta}} - \sin^2\theta$

$= 1 - \sin^2\theta$

We know $\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$= \cos^2\theta$

Tricks to Prove Identities:

There are multiple different strategies to prove identities, here are a few that might help you out!

1. Start with the side that looks the most complicated.

2. Change everything to cosθ and sinθ using the Quotient and Reciprocal Identities.

3. Look to apply the Pythagorean identity if you have a $\sin^2\theta$ or $\cos^2\theta$.

If you are still stuck after the first tricks look for the following:

1. Find a common denominator

2. Expand

3. Factor (either a common factor or decomposition)

Simplify

a) $\tan\theta + \cot\theta$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin\theta(\sin\theta)}{\cos\theta(\sin\theta)} + \frac{\cos\theta(\cos\theta)}{\sin\theta(\cos\theta)}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

where $\sin\theta \neq 0$
$\cos\theta \neq 0$

b) $\dfrac{1 - \sin^2\theta}{1 - \sin\theta}$

Factor by difference of squares

$$= \frac{(1+\sin\theta)\cancel{(1-\sin\theta)}}{\cancel{(1-\sin\theta)}}$$

$$= 1+\sin\theta$$

where $\sin\theta \neq 1$

Prove the following identities

a) $\tan\theta + \dfrac{1}{\tan\theta} = \dfrac{1}{\sin\theta\cos\theta}$ b) $\dfrac{1}{1+\cos\theta} + \dfrac{1}{1-\cos\theta} = \dfrac{2}{\sin^2\theta}$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

where $\sin\theta \neq 0$, $\cos\theta \neq 0$

$$= \frac{1}{\sin\theta\cos\theta} = RS$$

Prove the following identities

a) $\tan\theta + \dfrac{1}{\tan\theta} = \dfrac{1}{\sin\theta\cos\theta}$ b) $\dfrac{1}{1+\cos\theta} + \dfrac{1}{1-\cos\theta} = \dfrac{2}{\sin^2\theta}$

$$\frac{1(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} + \frac{1(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1-\cos\theta + 1 + \cos\theta}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{2}{1-\cos^2\theta}$$

$\cos\theta \neq \pm 1$
$\sin\theta \neq 0$

$$= \frac{2}{\sin^2\theta} = RS$$

Prove the following identities

a) $\tan\theta = \dfrac{\sin\theta + \sin^2\theta}{(\cos\theta)(1 + \sin\theta)}$ b) $\dfrac{\sin^2\theta}{1 - \cos\theta} = 1 + \cos\theta$

$$= \frac{\sin\theta\,\cancel{(1+\sin\theta)}}{\cos\theta\,\cancel{(1+\sin\theta)}}$$

$$= \frac{\sin\theta}{\cos\theta}$$

where $\cos\theta \neq 0$, $\sin\theta \neq -1$

$$= \tan\theta = LS$$

Prove the following identities

a) $\tan\theta = \dfrac{\sin\theta + \sin^2\theta}{(\cos\theta)(1 + \sin\theta)}$ b) $\dfrac{\sin^2\theta}{1 - \cos\theta} = 1 + \cos\theta$

$$= \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$= \frac{(1+\cos\theta)\cancel{(1-\cos\theta)}}{\cancel{(1-\cos\theta)}}$$

$$= 1+\cos\theta = RS$$

where $\cos\theta \neq 1$