Solve Problems Involving Right Triangles

Lesson objectives

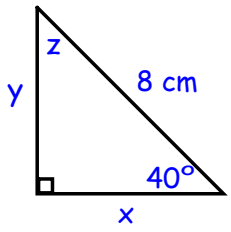
- I know how to choose the appropriate trigonometric ratio to solve a problem
- I know how to apply the primary trigonometric ratios to solve a word problem
- I know what angles of elevation and depression are

1.1 Lesson objectives | Teachers' notes | Lesson notes

MHR Page 380 #s 1, 5, 7, 11, 12, 15, 20, 21, & 23

Warm Up

Solve the triangles (determine the missing sides and angles).



$z = 180 - 90 - 40$

$z = 50°$

$\sin(40) = \frac{opp}{hyp}$

$\sin(40) = \frac{y}{8}$

$8\sin(40) = y$

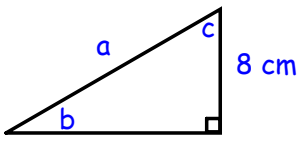
$5.1\text{ cm} = y$

$\cos(40) = \frac{adj}{hyp}$

$\cos(40) = \frac{x}{8}$

$8\cos(40) = x$

$6.1\text{ cm} = x$



$\tan(b) = \frac{opp}{adj}$

$\tan(b) = \frac{8}{12}$

$b = \tan^{-1}\left(\frac{8}{12}\right)$

$b = 33.7°$

$c = 180 - 90 - 33.7$

$c = 56.3°$

$a^2 = 8^2 + 12^2$

$a^2 = 64 + 144$

$a^2 = 208$

$a = \sqrt{208}$

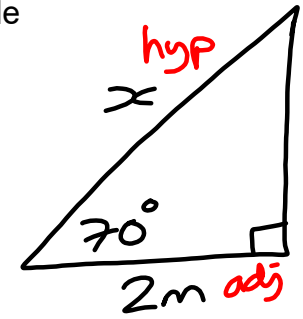
$a = 14.4\text{cm}$

Solving Word Problems

The key to solving word problems is to have a good diagram!

1. Sketch a diagram (include measurements)
2. Label the three sides and determine the reference angle
3. Choose the appropriate ratio using (Have/Need/Use).
4. Determine your missing information.
5. Write a concluding sentence.

Example
A ladder is placed 2 metres from a wall at an angle of 70°. How long is the ladder?



Have : adj, angle
Need : hyp
Use : cos

$\cos(\theta) = \frac{adj}{hyp}$

$\cos(70) = \frac{2}{x}$

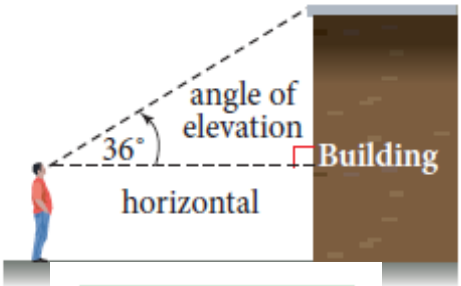
$\frac{x\cos(70)}{\cos 70} = \frac{2}{\cos 70}$

$x = 5.8$

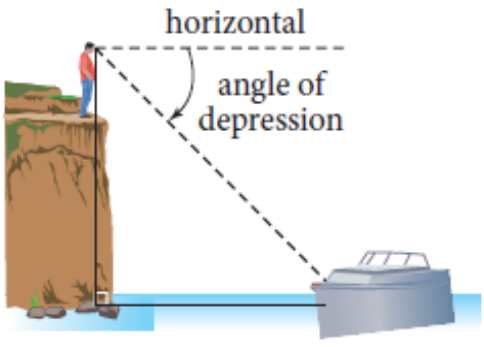
$\Rightarrow$ The ladder is 5.8 m long.

Angle Terminology

To talk about angles, we need to have a reference point. Sometimes, we use an **angle of elevation (inclination)** or an **angle of depression (declination)**.



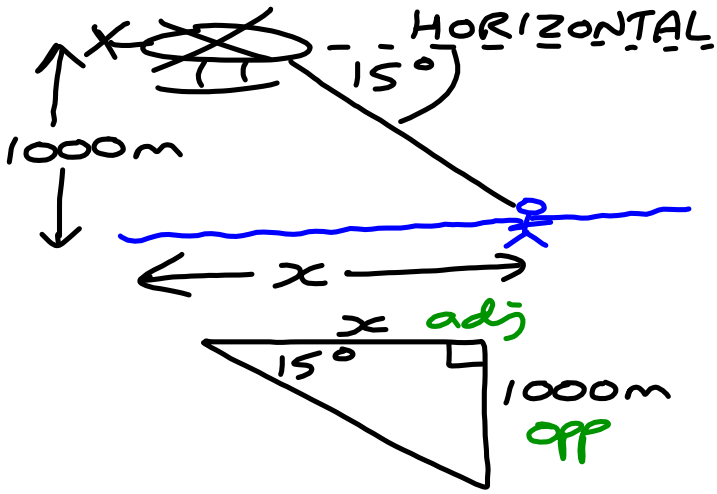
angle of elevation
- the angle between the horizontal and the line of sight up to an object
- also known as an angle of inclination



angle of depression
- the angle between the horizontal and the line of sight down to an object.

Example

A helicopter is flying at a height of 1000m and spots a person in the water at an angle of declination of 15°. What is the horizontal distance the helicopter must fly to be directly above the person?



Have : opp, angle
Need : adj
Use : TAN

$\tan(\theta) = \frac{opp}{adj}$

$\tan(15) = \frac{1000}{x}$

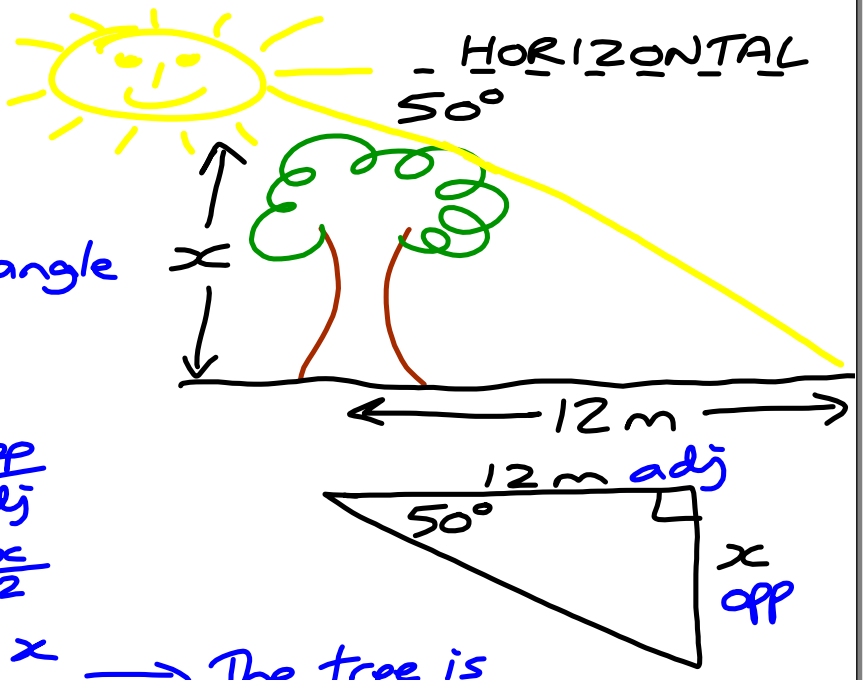
$\frac{x\tan(15)}{\tan(15)} = \frac{1000}{\tan(15)}$

$x = 3732.1$

$\Rightarrow$ Helicopter must fly a horizontal distance of 3732.1m to be above the person.

Example

The sun is shining at an angle of depression of 50°. It hits a tree and casts a shadow 12m long. What is the height of the tree?



Have : adj, angle
Need : opp
Use : TAN

$\tan(\theta) = \frac{opp}{adj}$

$\tan(50) = \frac{x}{12}$

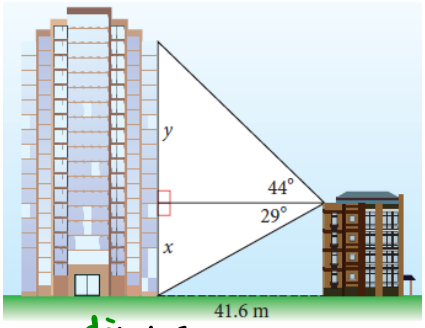
$12\tan(50) = x$

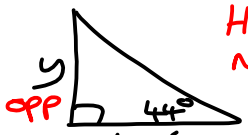
$14.3\text{m} = x$

$\Rightarrow$ The tree is 14.3m tall.

Find the Heights of Two Buildings

Two buildings are 41.6m apart. From the roof of the shorter building, the angle of elevation to the top of the taller building is 44° and the angle of depression to the base of the taller building is 29°. Find the heights of the buildings to the nearest tenth of a metre.





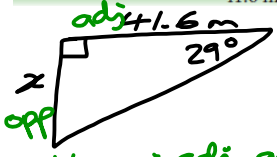
Have : adj, angle
Need : opp
Use : TAN

$\tan(\theta) = \frac{opp}{adj}$

$\tan(44) = \frac{y}{41.6}$

$41.6\tan(44) = y$

$40.2 = y$



Have : adj, angle
Need : opp
Use : TAN

$\tan(\theta) = \frac{opp}{adj}$

$\tan(29) = \frac{x}{41.6}$

$41.6\tan(29) = x$

$23.1 = x$

$\Rightarrow$ Small building is 23.1 m tall.

Taller building is $40.2 + 23.1 = 63.3$ m tall

A truck parked on the street is 56m from the base of the apartment building on the right. The angle of elevation from the truck to the top of the building is 40°. The parked truck is 49m from the base of the apartment building on the left. The angle of elevation from the truck to the top of the building is 50°. Which building is taller?

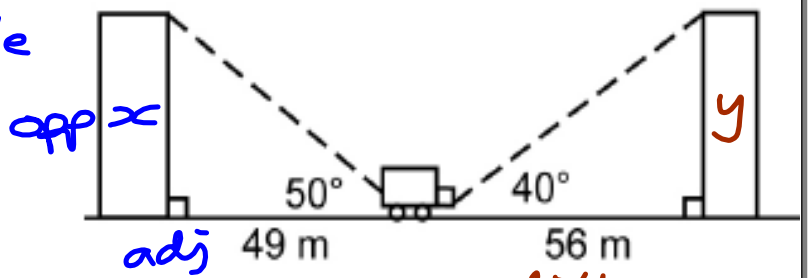
Have : adj, angle
Need : opp
Use : TAN

$\tan\theta = \frac{opp}{adj}$

$\tan(50) = \frac{x}{49}$

$49\tan(50) = x$

$58.4 = x$



Same conditions

$\Rightarrow \tan(40) = \frac{y}{56}$

$56\tan(40) = y$

$47.0 = y$

$\Rightarrow$ The building on the left is taller.

Example

For a bike ramp to be deemed "safe" it must have an angle of inclination of less than 20°. If Bunter builds a ramp that has a rise of 4m and covers a horizontal distance of 8m, is it "safe"?

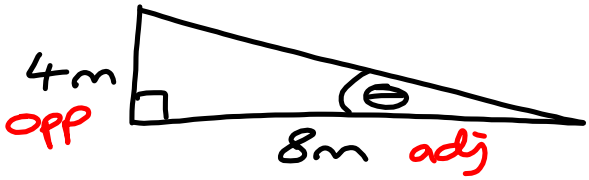
Have : opp, adj
Need : angle
Use : TAN

$\tan(\theta) = \frac{opp}{adj}$

$\tan(\theta) = \frac{4}{8}$

$\theta = \tan^{-1}\left(\frac{4}{8}\right)$

$\theta = 26.7°$



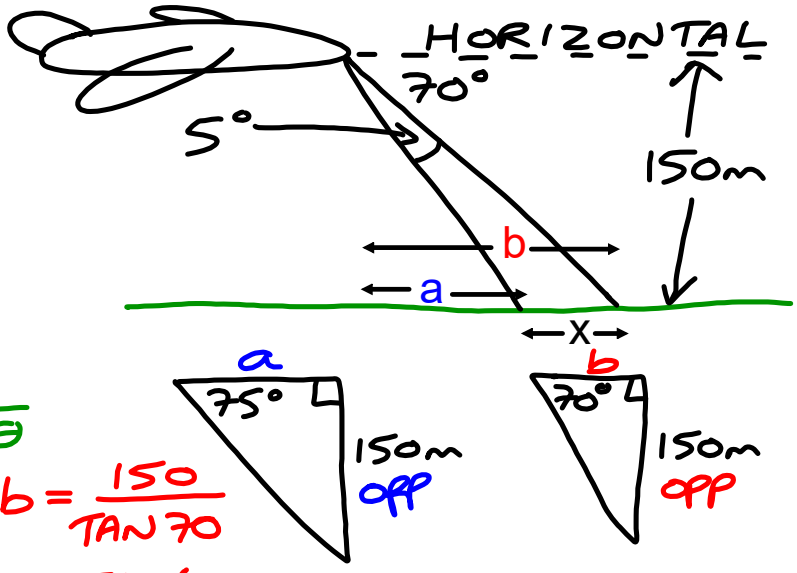
$\Rightarrow$ The angle of the ramp is greater than 20°, so it is deemed to be unsafe.

Example - Width of a Search Beam

An airplane is 150m above the ground and the search light is angled at 70° from the horizon. The beam spreads out at an angle of 5°. How much ground does the search beam cover?

Have : opp, angle
Need : adj
Use : TAN

$\Rightarrow adj = \frac{opp}{\tan\theta}$



$a = \frac{150}{\tan(75)}$

$a = 40.2$

$b = \frac{150}{\tan 70}$

$b = 54.6$

$\Rightarrow$ Search beam covers $54.6 - 40.2 = 14.4$ m (diameter)

Example

Suppose you are standing on a building 150 metres high. You look down to another building with your clinometer and measure the angle of depression to be 38°. You look to the top of the building and measure the angle of elevation to be 32°. How far away is the building? How high is the building?

Have : opp, angle

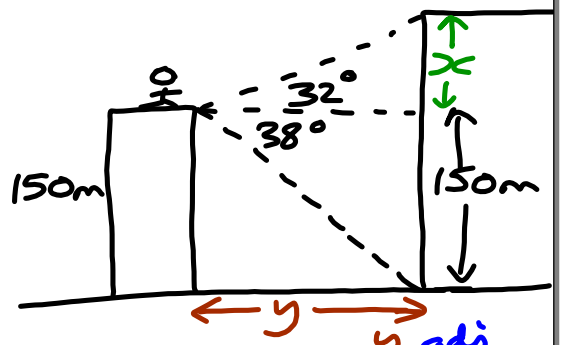
Need : adj

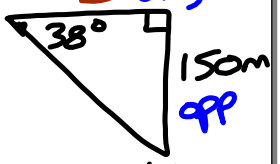
Use : TAN

$adj = \frac{opp}{TAN\theta}$

$y = \frac{150}{TAN(38)}$

$y = 192.0$





$opp = adj(TAN(\theta))$

$x = 192.0(TAN(32))$

$x = 120.0$

$\Longrightarrow$ Distance between buildings is 192.0 m. Height of taller building is 120.0 + 150 = 270.0 m