Exploring Trigonometric Ratios for Angles Greater than 90°

Lesson objectives

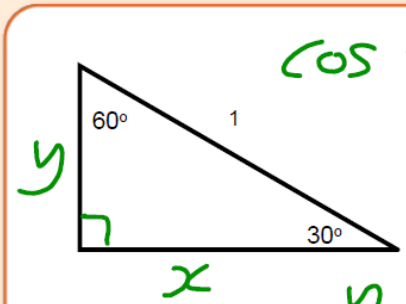
- I know how to draw the special angles in each quadrant of the unit circle
- I know how to extend the first quadrant of the unit circle to the other three
- I understand how the CAST rule works

1.1

Lesson objectives | Teachers' notes | Lesson notes

Nelson Page 300 #s 5, 6ace, 8abc, 9, 12 & 15

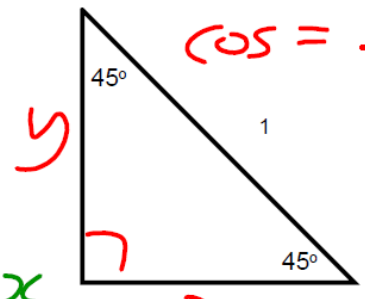
Find the other side lengths of the given triangles. Use exact values.



$\cos = \frac{A}{H}$

$\cos 60 = \frac{y}{1}$ $\cos 30 = \frac{x}{1}$

$\frac{1}{2} = y$ $\frac{\sqrt{3}}{2} = x$



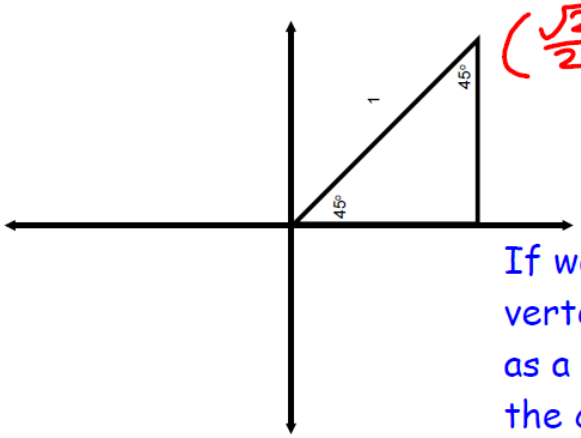
$\cos = \frac{A}{H}$

$\cos 45 = \frac{y}{1}$

$\frac{\sqrt{2}}{2} = y = x$

Unit Circle

We can transpose a triangle onto the Cartesian Plane.

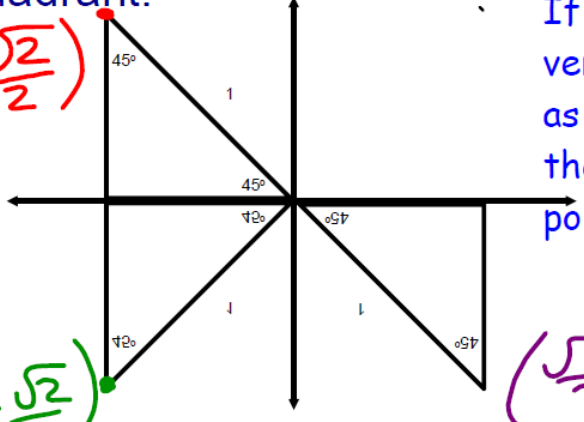


$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

If we look at the top vertex of the triangle as a point... what are the coordinates of that point?

Unit Circle

We can move the triangles around so there is one in each quadrant.



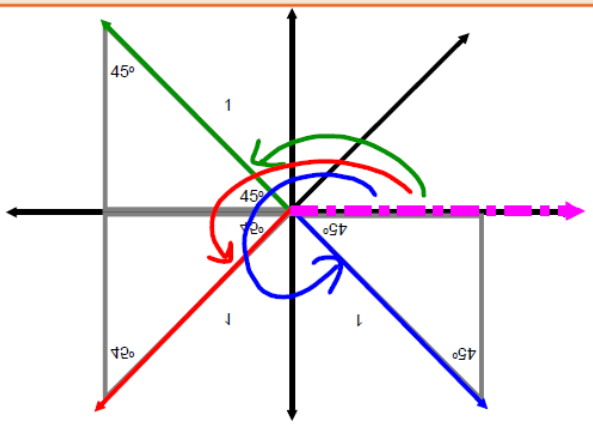
$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

If we look at the top vertex of each triangle as a point... what are the coordinates of each point?

Let's look at the measurement of each angle from the same starting point...



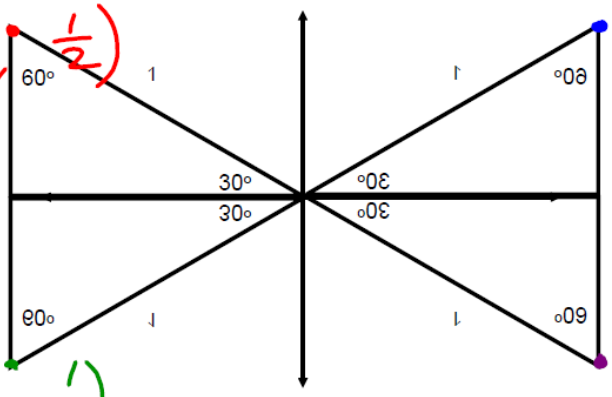
$135^\circ \left(\begin{matrix}180\\-45\end{matrix}\right)$

$225^\circ \left(\begin{matrix}180\\+45\end{matrix}\right)$

$315^\circ \left(\begin{matrix}360\\-45\end{matrix}\right)$

Unit Circle

Let's do the same for a 30° angle!



$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

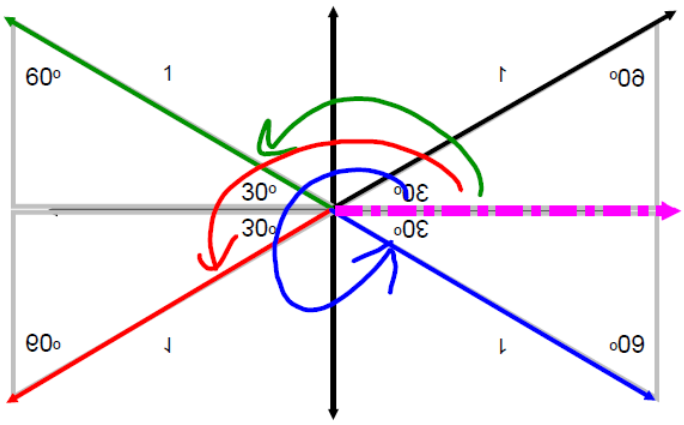
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

If we look at the vertex of the triangles as a point... what are the coordinates of each point?

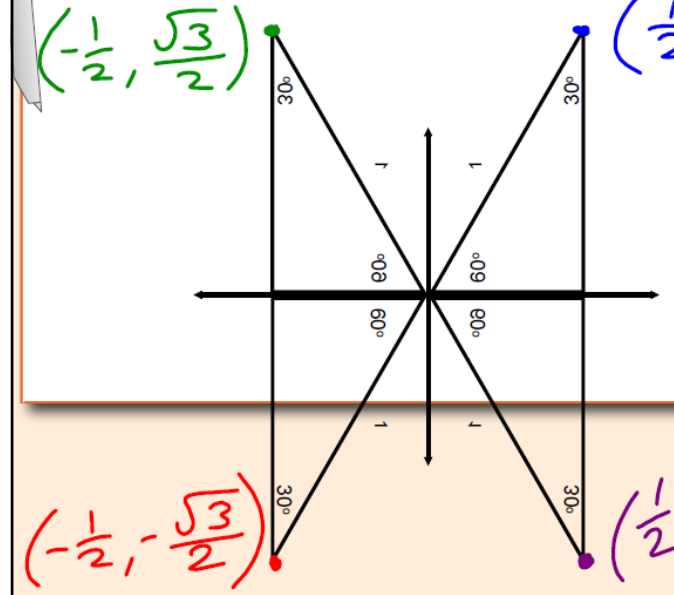
Let's look at the measurement of each angle from the same starting point...

Let's do the same for a 30° angle!

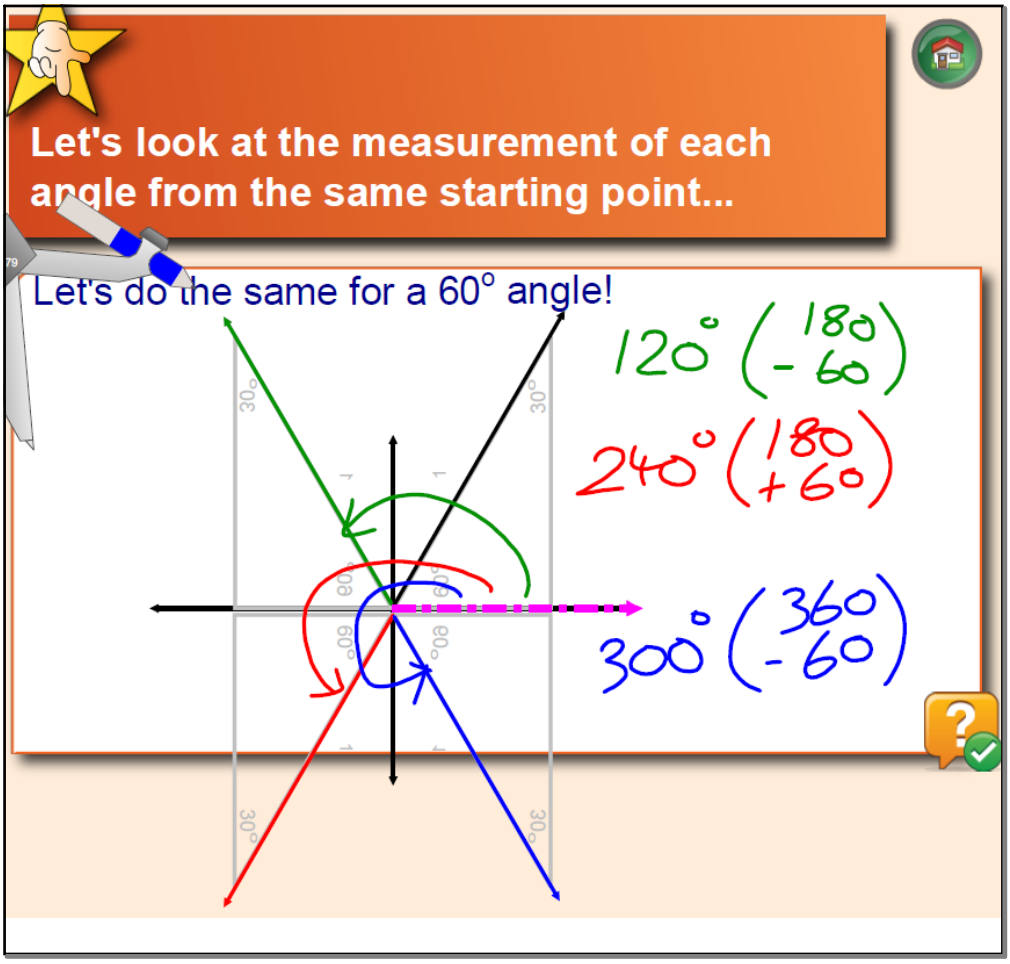


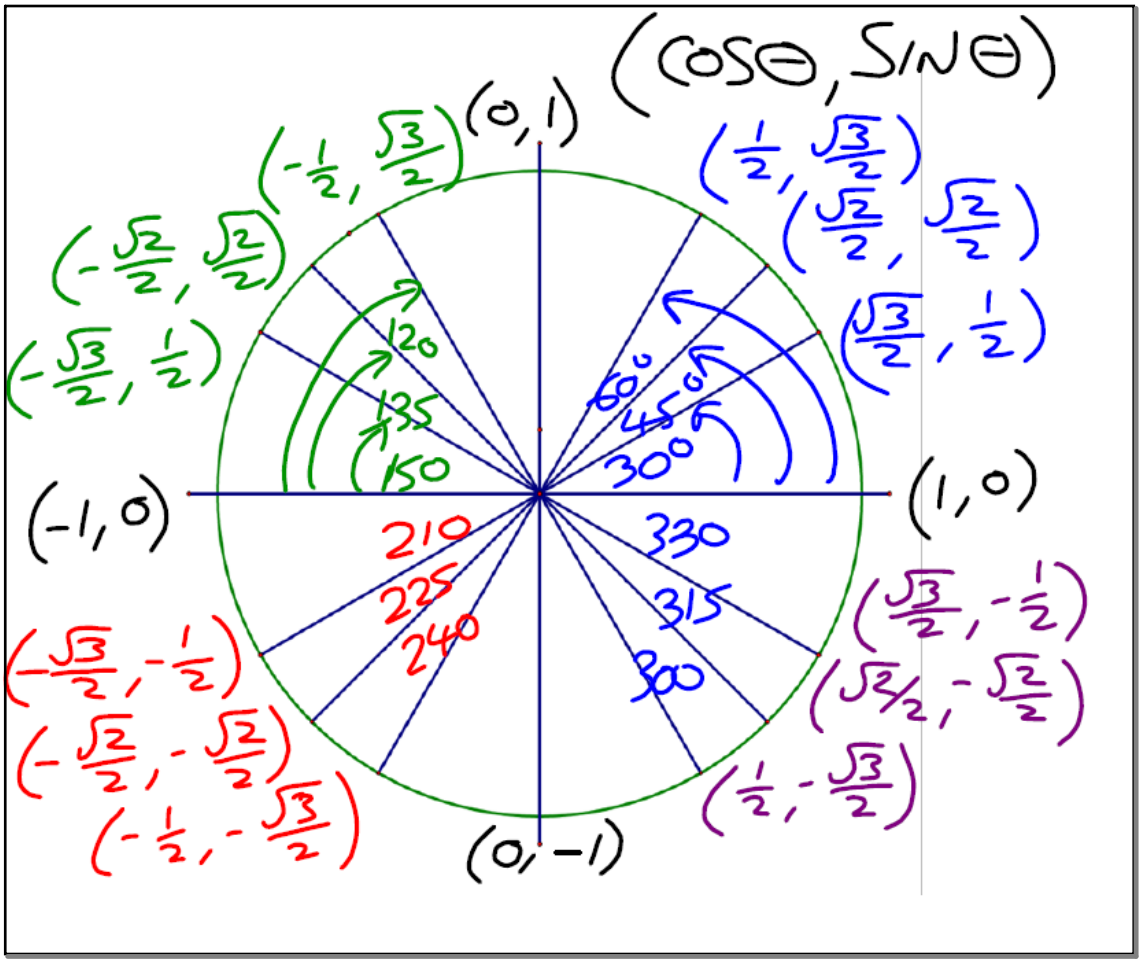
Unit Circle

Let's do the same for a 60° angle!



If we look at the vertex of the triangles as a point... what are the coordinates of each point?


Let's look at the measurement of each angle from the same starting point...
Let's do the same for a 60° angle!
120° (180 - 60)
240° (180 + 60)
300° (360 - 60)


(cosθ, sinθ)
(0,1)
(1,0)
(-1,0)
(0,-1)
(1/2, √3/2)
(√2/2, √2/2)
(√3/2, 1/2)
(-1/2, √3/2)
(-√2/2, √2/2)
(-√3/2, 1/2)
(√3/2, -1/2)
(√2/2, -√2/2)
(1/2, -√3/2)
(-√3/2, -1/2)
(-√2/2, -√2/2)
(-1/2, -√3/2)
60°
45°
30°
120
135
150
210
225
240
330
315
300

Determining the value of tangent

We know the following:

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

But is there a relationship between sine and cosine?

$$\sin\theta \div \cos\theta = \frac{opp}{hyp} \div \frac{adj}{hyp}$$

$$= \frac{opp}{\cancel{hyp}} \times \frac{\cancel{hyp}}{adj}$$

$$= \frac{opp}{adj} = \tan\theta$$

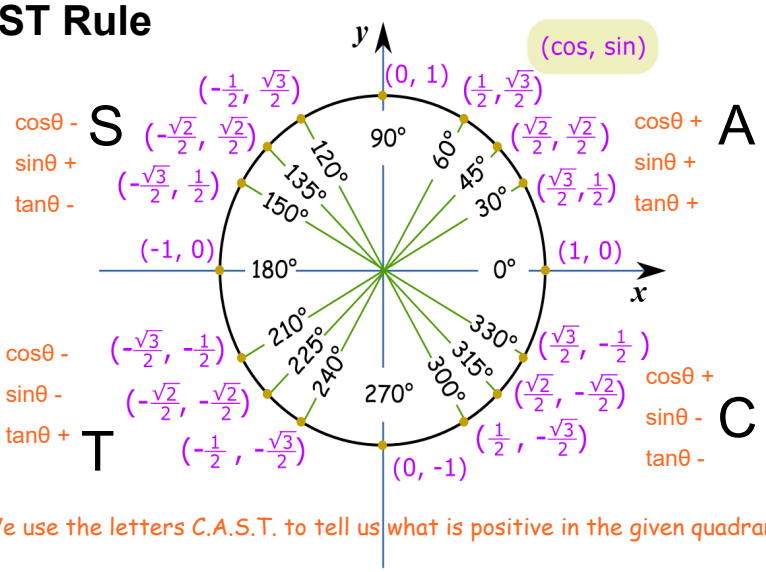
Unit Circle - More Patterns

Given that the x-value of the coordinate on the unit circle represents cos and the y-value of the coordinate on the unit circle represents sine.

List the quadrants where each are positive:

sine	cosine	tangent
quad 1	quad 1	quad 1 (two positive = +)
quad 2	quad 4	quad 3 (two negatives = +)

CAST Rule



We use the letters C.A.S.T. to tell us what is positive in the given quadrant

C - cos A - all S - sin T - tan

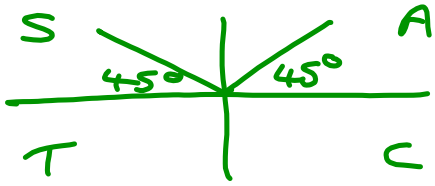
Where the ratio is not positive it is negative!

We can summarize the positive/negative values using the C.A.S.T. Rule.
We are talking about the **value of the ratio** not the angle!

$\sin(\theta) = \sqrt{2}/2$ $\cos(\theta) = -1/2$ $\tan(\theta) = 0.7$

$\theta = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\theta = 45°$

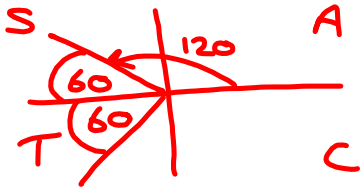


$\Rightarrow \theta = 180 - 45$

$\theta = 135°$

$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$

$\theta = 120°$

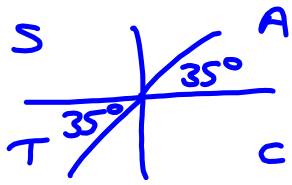


$\Rightarrow \theta = 180 + 60$

$\theta = 240°$

$\theta = \tan^{-1}(0.7)$

$\theta = 35°$



$\Rightarrow \theta = 180 + 35$

$\theta = 215°$