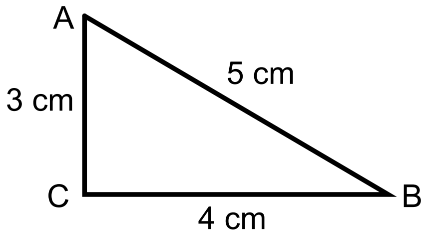
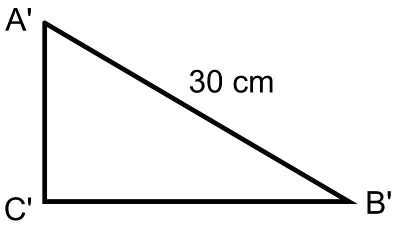
Solutions

1. A right triangle has side lengths 3 cm, 4 cm, and 5 cm.
   a) Draw the triangle.
   b) A similar triangle has hypotenuse 30 cm long. What is the scale factor?
   c) What are the lengths of the legs?

a)



b)



$$\text{Scale factor} = \frac{A'B'}{AB} = \frac{30}{5} = 6$$

c) Multiply the scale factor by the length of each leg

A'C' = AC x 6

= 3 x 6

= 18 cm

B'C' = BC x 3

= 4 x 6

= 24 cm

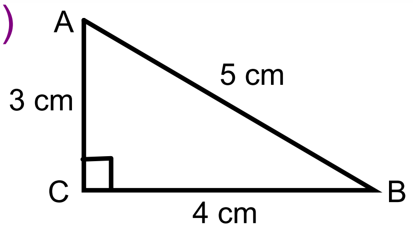
Note: Scale factors do not have any units

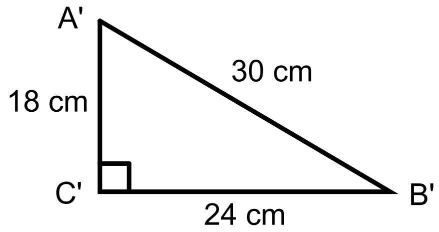
1. A right triangle has side lengths 3 cm, 4 cm, and 5 cm.
2. Refer to question 1.
   a) Find the area of each triangle.
   b) How are these areas related?
   c) How do the areas help to confirm that the triangles are similar?

Recall: 3, 4, 5 triangles are right triangles (Pythagorean theorem) and

Area = (base x height) ÷ 2

a)





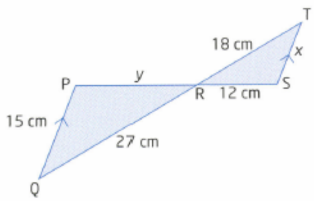
Area ABC = (4 x 3) ÷ 2
= 12 ÷ 2
= 6 cm^2

Area A'B'C' = (24 x 18) ÷ 2
= 432 ÷ 2
= 216 cm^2

b) The area A'B'C' is 36 times larger (216 ÷ 6) than ABC.

c) The area of similar triangles are related by the scale factor squared. Scale factor = 6, so scale factor squared = 6^2 = 36.

5. a) Show why △PQR is similar to △STR.
   b) Find the lengths x and y.



a) <PRQ = <TRS. These are opposite angles.

<QPR = <TSR. These are equal because TS is parallel to PQ.

<PQR = <RTS. Two angles are equal in each triangle, so the third pair must also be equal.

b) $\frac{TS}{PQ} = \frac{RS}{PR} = \frac{RT}{QR} = \frac{18}{27} = \frac{2}{3}$

Using $\frac{TS}{PQ} = \frac{RT}{QR}$ and $\frac{PR}{RS} = \frac{QR}{RT}$

$\frac{x}{15} = \frac{18}{27}$ $\qquad$ $\frac{y}{12} = \frac{27}{18}$

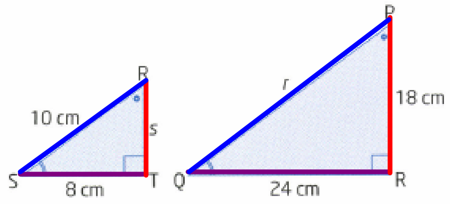
x = 15(18 ÷ 27) $\qquad$ y = 12(27 ÷ 18)

x = 10 cm $\qquad$ y = 18 cm

Note: It is easier to solve when you set the proportion up with your unknown in the top left .

6. The triangles in each pair are similar. Find the unknown side lengths.

b)



Left fraction is Unknown ÷ corresponding side

Right fraction is corresponding ÷ corresponding for the known pair

Keep the sides from one triangle on the top and the sides from the other triangle on the bottom

Using $\frac{RT}{PR} = \frac{ST}{QR}$ and $\frac{PQ}{RS} = \frac{QR}{ST}$

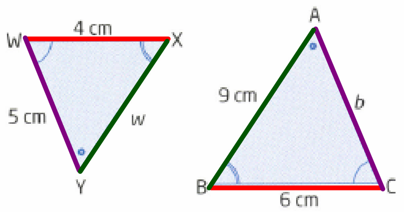
$\frac{s}{18} = \frac{8}{24}$ $\frac{r}{10} = \frac{24}{8}$

$s = 18(8 \div 24)$ $r = 10(24 \div 8)$

$s = 6$ cm $r = 30$ cm

6. The triangles in each pair are similar. Find the unknown side lengths.

c)



Left fraction is Unknown ÷ corresponding side

Right fraction is corresponding ÷ corresponding for the known pair

Keep the sides from one triangle on the top and the sides from the other triangle on the bottom

Using $\frac{AC}{WY} = \frac{BC}{XW}$ and $\frac{XY}{AB} = \frac{XW}{BC}$

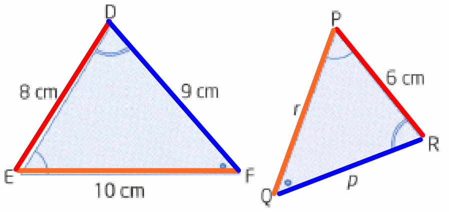
$\frac{b}{5} = \frac{6}{4}$ $\frac{w}{9} = \frac{4}{6}$

$b = 5(6 \div 4)$ $w = 9(4 \div 6)$

$b = 7.5$ cm $w = 6$ cm

6. The triangles in each pair are similar. Find the unknown side lengths.

d)



Left fraction is Unknown ÷ corresponding side

Right fraction is corresponding ÷ corresponding for the known pair

Keep the sides from one triangle on the top and the sides from the other triangle on the bottom

Using $\frac{QR}{DF} = \frac{PR}{DE}$ and $\frac{QP}{DF} = \frac{PR}{DE}$

$\frac{p}{9} = \frac{6}{8}$

$p = 9(6 \div 8)$

$p = 6.75$ cm

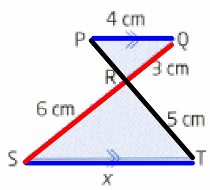
$\frac{r}{10} = \frac{6}{8}$

$r = 10(6 \div 8)$

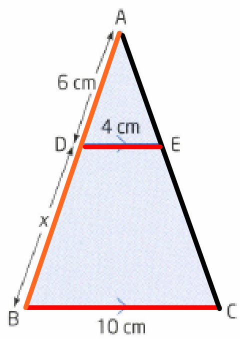
$r = 7.5$ cm

7. Find the length of x in each.

a)



b)



Using $\frac{ST}{PQ} = \frac{SR}{RQ}$

$\frac{x}{4} = \frac{6}{3}$

$x = 4(6 \div 3)$

$x = 8$ cm

and $\frac{AB}{AD} = \frac{BC}{DE}$

$\frac{x + 6}{6} = \frac{10}{4}$

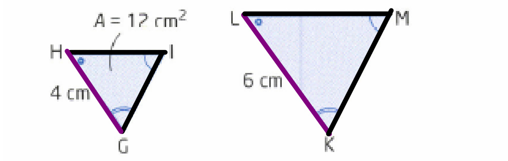
$x + 6 = 6(10 \div 4)$

$x + 6 = 15$

$x = 15 - 6$

$x = 9$ cm

8. c) △GHI ~ △KLM. Find the area of △KLM.



Scale factor = $\frac{KL}{GH} = \frac{6}{4}$

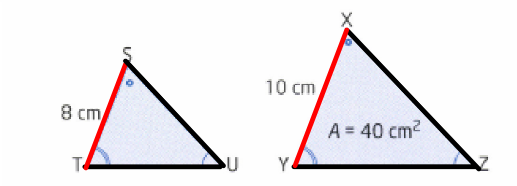
Area of KLM = Area of GHI x scale factor squared

Area = $12(6 \div 4)^2$

Area = 12(2.25)

Area = 27 cm^2

8. d) △STU ~ △XYZ. Find the area of △STU.



Scale factor = $\frac{ST}{XY} = \frac{8}{10}$

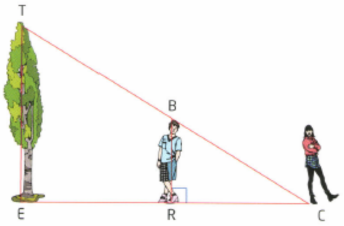
Area of STU = Area of XYZ x scale factor squared

Area = $40(8 \div 10)^2$

Area = 40(0.64)

Area = 25.6 cm^2

9. To measure the height of a tree, Cynthia has her little brother, BR, stand so that the tip of his shadow coincides with the tip of the tree's shadow, at point C.



Cynthia's brother, who is 1.2 m tall, is 4.2 m from Cynthia, who is standing at C, and 6.5 m from the base of the tree. Find the height of the tree, TE.

Using $\frac{TE}{EC} = \frac{BR}{RC}$

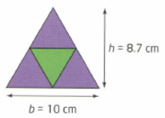
$$\frac{TE}{4.2 + 6.5} = \frac{1.2}{4.2}$$

$TE = 10.7(1.2 \div 4.2)$

$x = 3.06$ m

The height of the tree is 3.1 metres

12. Melanie is designing a crest for her hockey team, the Trigazoids. Her prototype consists of four congruent equilateral triangles.



a) What is the total area of this crest?

b) What is the area of
- the green section?
- the purple sections?

c) What is the area of a giant similar crest with base 30 cm?

d) What is the height of a similar crest with area 500 cm²?

a) Area = (base x height) ÷ 2

Area = (10 x 8.7) ÷ 2

Area = 43.5 cm^2

b) $Area_G$ = 1/4 x 43.5

= 10.875 cm^2

$Area_P$ = 3/4 x 43.5

= 32.625 cm^2

c) Scale factor = 30 ÷ 10 = 3

New area = original area x scale factor squared

New area = 43.5 x 3^2

New area = 391.5 cm^2

d) New area = 500 cm^2 Original area = 43.5 cm^2

Scale factor squared = new ÷ old

= 500 ÷ 43.5

= 11.49....

S.F. = $\sqrt{11.49...}$

= 3.39...

New height = Old height x Scale Factor

= 8.7 x 3.39...

= 29.5 cm

19. The areas of two similar triangles are $72\ cm^2$ and $162\ cm^2$. What is the ratio of the lengths of their corresponding sides?

Scale factor squared = large area ÷ small area

= 162 ÷ 72

= 2.25

Scale factor = $\sqrt{2.25}$

= 1.5

Therefore the ratio of corresponding sides for

small : large

= 1 : 1.5 (can't leave ratios as decimals)

= 2 : 3