Solutions

4. Determine the exact value of each trigonometric expression.

K **a)** $\sin 30° \times \tan 60° - \cos 30°$

$= \frac{1}{2} \times \sqrt{3} - \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$

$= 0$

b) $2\cos 45° \times \sin 45°$

$= 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= 2\left(\frac{2}{4}\right)$

$= \frac{4}{4} = 1$

c) $\tan^2 30° - \cos^2 45°$

$= \left(\frac{\sqrt{3}}{3}\right)^2 - \left(\frac{\sqrt{2}}{2}\right)^2$

$= \frac{3}{9} - \frac{2}{4}$

$= \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$

d) $1 - \frac{\sin 45°}{\cos 45°}$

$= 1 - \left(\frac{\sqrt{2}}{2}\right) \div \left(\frac{\sqrt{2}}{2}\right)$

$= 1 - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{2}{\sqrt{2}}\right)$

$= 1 - 1$

$= 0$

5. Using exact values, show that $\sin^2\theta + \cos^2\theta = 1$ for each angle.

a) $\theta = 30°$ b) $\theta = 45°$ c) $\theta = 60°$

$= \sin^2 30 + \cos^2 30$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{4} + \frac{3}{4}$

$= 1$

$= \sin^2 45 + \sin^2 45$

$= \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2$

$= \frac{2}{4} + \frac{2}{4}$

$= \frac{4}{4}$

$= 1$

$= \sin^2 60 + \cos^2 60$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} + \frac{1}{4}$

$= 1$

6. Using exact values, show that $\frac{\sin\theta}{\cos\theta} = \tan\theta$ for each angle.

a) $\theta = 30°$ b) $\theta = 45°$ c) $\theta = 60°$

$\frac{\sin 30}{\cos 30}$

$= \frac{1}{2} \div \frac{\sqrt{3}}{2}$

$= \frac{1}{2} \times \frac{2}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3} = \tan 30$

$\frac{\sin 45}{\cos 45}$

$= \frac{\sqrt{2}}{2} \div \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{2} \times \frac{2}{\sqrt{2}}$

$= 1 = \tan 45$

$\frac{\sin 60}{\cos 60}$

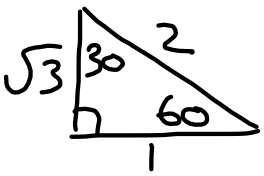
$= \frac{\sqrt{3}}{2} \div \frac{1}{2}$

$= \frac{\sqrt{3}}{2} \times \frac{2}{1}$

$= \sqrt{3} = \tan 60$

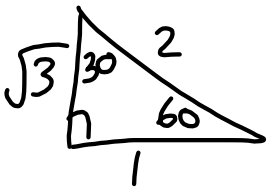
7. Using the appropriate special triangle, determine θ if $0° \le \theta \le 90°$.

a) $\sin\theta = \frac{\sqrt{3}}{2}$



$\theta = 60°$

b) $\frac{\sqrt{3}\tan\theta}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

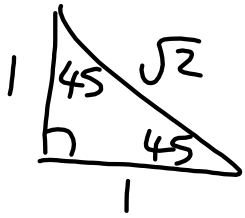


$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = 30°$

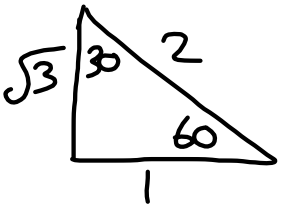
c) $\frac{2\sqrt{2}\cos\theta}{2\sqrt{2}} = \frac{2}{2\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}}$



$\theta = 45°$

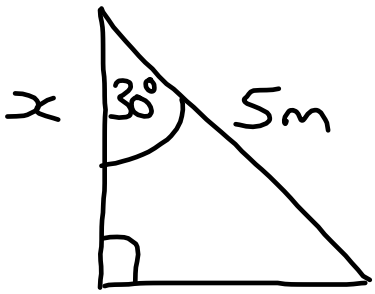
d) $\frac{2\cos\theta}{2} = \frac{\sqrt{3}}{2}$



$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = 30°$

8. A 5 m stepladder propped against a classroom wall forms an angle of 30° with the wall. Exactly how far is the top of the ladder from the floor? Express your answer in radical form. What assumption did you make?



$\cos\theta = \frac{adj}{hyp}$

$\cos 30 = \frac{x}{5}$

$\frac{\sqrt{3}}{2} = \frac{x}{5}$

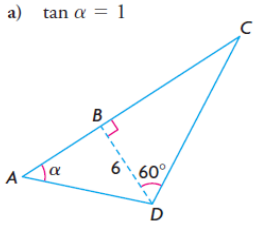
$\frac{5\sqrt{3}}{2} = x$

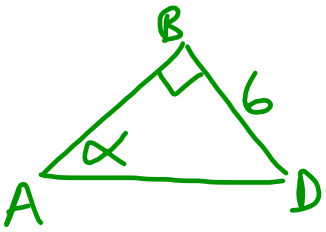
Height of $\frac{5\sqrt{3}}{2}$ metres

Assume the building and ground form a right angle

11. Determine the exact area of each large triangle.

a) $\tan\alpha = 1$



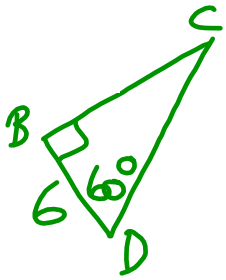


$\tan\alpha = 1$

$\tan\alpha = \frac{AB}{6}$

$1 = \frac{AB}{6}$

$6 = AB$



$\tan 60 = \frac{BC}{6}$

$6\tan 60 = BC$

$6\sqrt{3} = BC$

$\text{Area} = \frac{AC \times BD}{2}$

$= \frac{(6+6\sqrt{3})(6)}{2}$

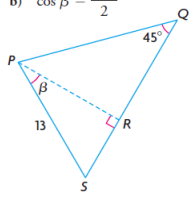
$= \frac{36+36\sqrt{3}}{2}$

$= 18+18\sqrt{3}$

$= 18(1+\sqrt{3})$ square units

11. Determine the exact area of each large triangle.

b) $\cos\beta = \frac{\sqrt{3}}{2}$



$\cos\beta = \frac{\sqrt{3}}{2}$

$\cos\beta = \frac{PR}{13}$

$\frac{\sqrt{3}}{2} = \frac{PR}{13}$

$\frac{13\sqrt{3}}{2} = PR$

$(30, 60, 90)$

$\sin\beta = \frac{1}{2}$

$\sin\beta = \frac{RS}{13}$

$\frac{1}{2} = \frac{RS}{13}$

$13/2 = RS$

$\tan 45 = \frac{RQ}{PR}$

$1 = \frac{RQ}{\frac{13\sqrt{3}}{2}}$

$\frac{13\sqrt{3}}{2} = RQ$

$\text{Area} = \frac{SQ \times PR}{2}$

$= \frac{\left(\frac{13}{2} + \frac{13\sqrt{3}}{2}\right)\left(\frac{13\sqrt{3}}{2}\right)}{2}$

$= \frac{\frac{169\sqrt{3}}{4} + \frac{169(\sqrt{3})(\sqrt{3})}{4}}{2}$

$= \frac{1}{2}\left(\frac{169\sqrt{3} + 169(3)}{4}\right)$

$= \frac{169(\sqrt{3}+3)}{8}$ square units