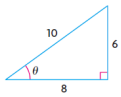
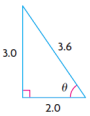
Solutions

5. a) For each triangle, calculate csc θ, sec θ, and cot θ.
K b) For each triangle, use one of the reciprocal ratios from part (a) to determine θ to the nearest degree.

i)



iii)



$\csc\theta = \frac{Hyp}{Opp}$

$= \frac{10}{6} = \frac{5}{3}$

$\sec\theta = \frac{Hyp}{Adj}$

$= \frac{10}{8} = \frac{5}{4}$

$\cot\theta = \frac{Adj}{Opp}$

$= \frac{8}{6} = \frac{4}{3}$

$\csc\theta = \frac{5}{3}$

$\frac{1}{\sin\theta} = \frac{5}{3}$

$\sin\theta = \frac{3}{5}$

$\theta = \sin^{-1}\left(\frac{3}{5}\right)$

$\theta = 37°$

$\csc\theta = \frac{Hyp}{Opp}$

$= \frac{3.6}{3.0} = 1.2$

$\sec\theta = \frac{Hyp}{Opp}$

$= \frac{3.6}{2.0} = 1.8$

$\cot\theta = \frac{Adj}{Opp}$

$= \frac{2.0}{3.0} = \frac{2}{3}$

$\sec\theta = 1.8$

$\frac{1}{\cos\theta} = 1.8$

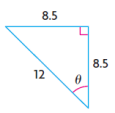
$\cos\theta = \frac{1}{1.8}$

$\theta = \cos^{-1}\left(\frac{5}{9}\right)$

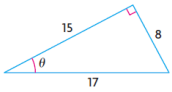
$\theta = 56°$

5. a) For each triangle, calculate csc θ, sec θ, and cot θ.
K b) For each triangle, use one of the reciprocal ratios from part (a) to determine θ to the nearest degree.

ii)



iv)



$\csc\theta = \frac{Hyp}{Opp}$

$= \frac{12}{8.5} = 1.4118$

$\sec\theta = \frac{Hyp}{Adj}$

$= \frac{12}{8.5} = 1.4118$

$\cot\theta = \frac{Adj}{Opp}$

$= \frac{8.5}{8.5} = 1$

$\cot\theta = 1$

$\frac{1}{\tan\theta} = 1$

$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$\theta = 45°$

$\csc\theta = \frac{Hyp}{Opp}$

$= \frac{17}{8} = 2.125$

$\sec\theta = \frac{Hyp}{Adj}$

$= \frac{17}{15} = 1.1333$

$\cot\theta = \frac{Adj}{Opp}$

$= \frac{8}{15} = 0.5333$

$\csc\theta = 2.125$

$\frac{1}{\sin\theta} = 2.125$

$\sin\theta = \frac{8}{17}$

$\theta = \sin^{-1}\left(\frac{8}{17}\right)$

$\theta = 28°$

6. Determine the value of θ to the nearest degree.

a) $\cot\theta = 3.2404$ c) $\sec\theta = 1.4526$

b) $\csc\theta = 1.2711$ d) $\cot\theta = 0.5814$

a) $\cot\theta = 3.2404$

$\frac{1}{\tan\theta} = 3.2404$

$\tan\theta = 0.3086$

$\theta = \tan^{-1}(0.3086)$

$\theta = 17°$

b) $\csc\theta = 1.2711$

$\frac{1}{\sin\theta} = 1.2711$

$\sin\theta = 0.7867$

$\theta = \sin^{-1}(0.7867)$

$\theta = 52°$

c) $\sec\theta = 1.4526$

$\frac{1}{\cos\theta} = 1.4526$

$\cos\theta = 0.6884$

$\theta = \cos^{-1}(0.6884)$

$\theta = 46°$

d) $\cot\theta = 0.5814$

$\frac{1}{\tan\theta} = 0.5814$

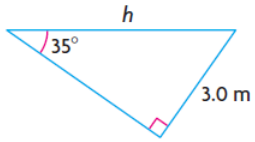
$\tan\theta = 1.7199$

$\theta = \tan^{-1}(1.7199)$

$\theta = 60°$

7. For each triangle, determine the length of the hypotenuse to the nearest tenth of a metre.

a)



Have: opp, angle

Need: hyp

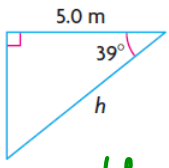
Use: $\sin\theta = \frac{opp}{hyp}$

$\sin 35 = \frac{3.0}{h}$

$\frac{h \sin 35}{\sin 35} = \frac{3.0}{\sin 35}$

$h = 5.2\,m$

b)



Have: adj, angle

Need: hyp

Use: $\cos\theta = \frac{adj}{hyp}$

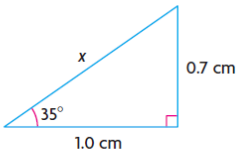
$\cos 39 = \frac{5.0}{h}$

$\frac{h \cos 39}{\cos 39} = \frac{5.0}{\cos 39}$

$h = 6.4\,m$

8. For each triangle, use two different methods to determine x to the nearest tenth of a unit.

a)



$\sin 35 = \frac{0.7}{x}$

$\frac{x \sin 35}{\sin 35} = \frac{0.7}{\sin 35}$

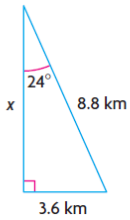
$x = 1.2\,cm$

$\cos 35 = \frac{1.0}{x}$

$\frac{x \cos 35}{\cos 35} = \frac{1.0}{\cos 35}$

$x = 1.2\,cm$

b)



$\cos 24 = \frac{x}{8.8}$

$8.8 \cos 24 = x$

$x = 8.0\,km$

$\tan 24 = \frac{3.6}{x}$

$\frac{x \tan 24}{\tan 24} = \frac{3.6}{\tan 24}$

$x = 8.1\,km$

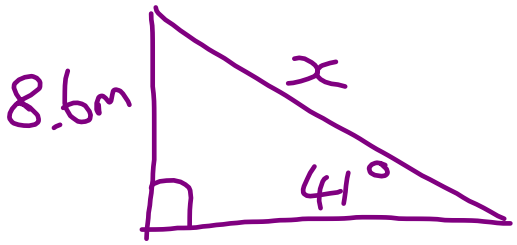
[Different due to rounding]

11. A kite is flying 8.6 m above the ground at an angle of elevation of 41°.

A Calculate the length of string, to the nearest tenth of a metre, needed to fly the kite using

a) a primary trigonometric ratio

b) a reciprocal trigonometric ratio



a) $\sin 41 = \frac{8.6}{x}$

$\frac{x \sin 41}{\sin 41} = \frac{8.6}{\sin 41}$

$x = 13.1\text{m}$

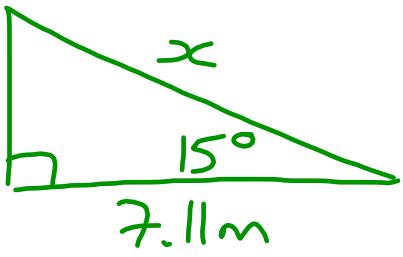
b) $\csc 41 = \frac{x}{8.6}$

$8.6 \csc 41 = x$

$\frac{8.6}{\sin 41} = x$

$x = 13.1\text{m}$

12. A wheelchair ramp near the door of a building has an incline of 15° and a run of 7.11 m from the door. Calculate the length of the ramp to the nearest hundredth of a metre.



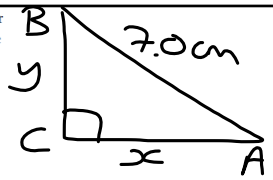
$\cos 15 = \frac{7.11}{x}$

$\frac{x \cos 15}{\cos 15} = \frac{7.11}{\cos 15}$

$x = 7.36\text{m}$

13. The hypotenuse, c, of right $\triangle ABC$ is 7.0 cm long. A trigonometric ratio for angle A is given for four different triangles. Which of these triangles has the greatest area? Justify your decision.

a) $\sec A = 1.7105$ c) $\csc A = 2.2703$
b) $\cos A = 0.7512$ d) $\sin A = 0.1515$



a) $\sec A = 1.7105$

$\cos A = \frac{1}{1.7105}$

$\cos A = \frac{x}{7.0}$

$\Rightarrow \frac{x}{7.0} = \frac{1}{1.7105}$

$x = \frac{7.0}{1.7105}$

$x = 4.09\text{cm}$

$y = \sqrt{7.0^2 - 4.09^2}$

$y = 5.68\text{cm}$

$\text{Area} = \frac{4.09 \times 5.68}{2}$

$= 11.62\text{cm}^2$

c) $\csc A = 2.2703$

$\sin A = \frac{1}{2.2703}$

$\sin A = \frac{y}{7.0}$

$\Rightarrow \frac{y}{7.0} = \frac{1}{2.2703}$

$y = \frac{7.0}{2.2703}$

$y = 3.08\text{cm}$

$x = \sqrt{7.0^2 - 3.08^2}$

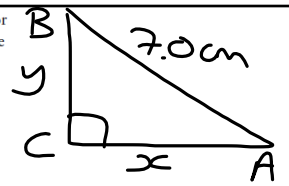
$x = 6.29\text{cm}$

$\text{Area} = \frac{3.08 \times 6.29}{2}$

$= 9.69\text{cm}^2$

13. The hypotenuse, c, of right $\triangle ABC$ is 7.0 cm long. A trigonometric ratio for angle A is given for four different triangles. Which of these triangles has the greatest area? Justify your decision.

a) $\sec A = 1.7105$ c) $\csc A = 2.2703$
b) $\cos A = 0.7512$ d) $\sin A = 0.1515$



b) $\cos A = 0.7512$

$\cos A = \frac{x}{7.0}$

$\Rightarrow \frac{x}{7.0} = 0.7512$

$x = 7.0(0.7512)$

$x = 5.26\text{cm}$

$y = \sqrt{7.0^2 - 5.26^2}$

$y = 4.62\text{cm}$

$\text{Area} = \frac{5.26 \times 4.62}{2}$

$= 12.15\text{cm}^2$

d) $\sin A = 0.1515$

$\sin A = \frac{y}{7.0}$

$\Rightarrow \frac{y}{7.0} = 0.1515$

$y = 7.0(0.1515)$

$y = 1.06\text{cm}$

$x = \sqrt{7.0^2 - 1.06^2}$

$x = 6.92\text{cm}$

$\text{Area} = \frac{1.06 \times 6.92}{2}$

$= 3.67\text{cm}^2$

$\Rightarrow$ Triangle B has the greatest area.