Unit 3 Inverse and Equivalent Functions Review

Page 132 #s 1, 3 - 5, 6aceg, 7ace, 9ac, 10ace, 11, 12bd, 13bd, 14bd, 15df & 16 and Handout 3.80 # 14

Review
- Inverse Functions
- Adding and Subtracting Polynomials
- Multiplying Polynomials
- Equivalent Expressions
- Reciprocal of a Linear Function
- Simplifying Rational Expressions
- Multiplying Rational Expressions
- Dividing Rational Expressions
- Adding and Subtracting Rational Expressions
- Solving Rational Expressions

6. Determine the inverse of each linear function by interchanging the variables.

b) $f(x) = 2 - x$

d) $f(x) = -\frac{1}{5}x - 2$

f) $f(x) = \frac{x-3}{4}$

$y = 2 - x$

$x = 2 - y$

$\dfrac{x-2}{-1} = \dfrac{-y}{-1}$

$2 - x = y$

$f^{-1}(x) = 2 - x$

$y = -\frac{1}{5}x - 2$

$x = -\frac{1}{5}y - 2$

$x + 2 = -\frac{1}{5}y$

$-5(x+2) = y$

$f^{-1}(x) = -5(x+2)$

$y = \dfrac{x-3}{4}$

$x = \dfrac{y-3}{4}$

$4x = y - 3$

$4x + 3 = y$

$f^{-1}(x) = 4x + 3$

Solutions

1. Simplify.

a) $(7x^2 - 2x + 1) + (9x^2 - 4x + 5) - (4x^2 + 6x - 7)$

$$= 7x^2 - 2x + 1 + 9x^2 - 4x + 5 - 4x^2 - 6x + 7$$

$$= 12x^2 - 12x + 13$$

b) $(7a^2 - 4ab + 9b^2) - (-a^2 + 2ab + 6b^2)$

$$= 7a^2 - 4ab + 9b^2 + a^2 - 2ab - 6b^2$$

$$= 8a^2 - 6ab + 3b^2$$

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3. Ms. Flanagan has three daughters: Astrid, Beatrice, and Cassandra. Today, January 1, their ages are, respectively,

$A(n) = -(n + 30) + (2n + 5)$

$B(n) = (7 - n) - (32 - 2n)$

$C(n) = (n - 26) - (n + 4) + (n - 3)$

All ages are expressed in years, and n represents Ms. Flanagan's age.

a) Are the daughters triplets? Explain.

b) Are any of them twins? Explain.

c) How old was Ms. Flanagan when Cassandra was born?

$A(n) = -(n+30) + (2n+5)$

$\quad\quad = -n - 30 + 2n + 5$

$\quad\quad = n - 25$

$B(n) = (7-n) - (32-2n)$

$\quad\quad = 7 - n - 32 + 2n$

$\quad\quad = n - 25$

$C(n) = (n-26) - (n+4) + (n-3)$

$\quad\quad = n - 26 - n - 4 + n - 3$

$\quad\quad = n - 33$

a) No. Not all the same age.

b) Probably. Astrid and Beatrice are the same age.

c) $C(n) = 0$

$\quad n - 33 = 0$

$\quad n = 33$

4. Expand and simplify.
 a) $-3(7x - 5)(4x - 7)$
 b) $-(y^2 - 4y + 7)(3y^2 - 5y - 3)$

a) $-3(7x-5)(4x-7)$

$= -3(28x^2 - 49x - 20x + 35)$

$= -3(28x^2 - 69x + 35)$

$= -84x^2 + 207x - 105$

b) $-(y^2 - 4y + 7)(3y^2 - 5y - 3)$

$= (-y^2 + 4y - 7)(3y^2 - 5y - 3)$

	$3y^2$	$-5y$	-3
$-y^2$	$-3y^4$	$5y^3$	$3y^2$
$4y$	$12y^3$	$-20y^2$	$-12y$
-7	$-21y^2$	$35y$	21

$= -3y^4 + 17y^3$
$ -38y^2 + 23y$
$ + 21$

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4. Expand and simplify.
 c) $2(a + b)^3$
 d) $3(x^2 - 2)^2(2x - 3)^2$

$2(a+b)(a+b)(a+b)$

$= (2a + 2b)(a^2 + ab + ab + b^2)$

$= (2a + 2b)(a^2 + 2ab + b^2)$

	$2a$	$2b$
a^2	$2a^3$	$2a^2b$
$2ab$	$4a^2b$	$4ab^2$
b^2	$2ab^2$	$2b^3$

$= 2a^3 + 6a^2b + 6ab^2$
$ + 2b^3$

$3(x^2-2)(x^2-2)(2x-3)$
$ (2x-3)$

$= (3x^2 - 6)(x^2 - 2)$
$ (4x^2 - 6x - 6x + 9)$

$= (3x^4 - 6x^2 - 6x^2 + 12)$
$ (4x^2 - 12x + 9)$

$= (3x^4 - 12x^2 + 12)$
$ (4x^2 - 12x + 9)$

	$3x^4$	$-12x^2$	12
$4x^2$	$12x^6$	$-48x^4$	$48x^2$
$-12x$	$-36x^5$	$144x^3$	$-144x$
9	$27x^4$	$-108x^2$	108

$= 12x^6 - 36x^5 - 21x^4 + 144x^3 - 60x^2 - 144x + 108$

5. The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$.
   Determine the volume of the cone in simplified form if the radius is increased by x and the height is increased by $2x$.

$$V = \frac{1}{3}\pi r^2 h$$

$$r \rightarrow r + x$$
$$h \rightarrow h + 2x$$

$$V = \frac{1}{3}\pi (r+x)^2 (h+2x)$$

$$V = \frac{1}{3}\pi (r+x)(r+x)(h+2x)$$

$$V = \frac{1}{3}\pi (r^2 + rx + rx + x^2)(h+2x)$$

$$V = \frac{1}{3}\pi (r^2 + 2rx + x^2)(h+2x)$$

$$V = \frac{1}{3}\pi (r^2 h + 2rxh + x^2 h + 2xr^2 + 4rx^2 + 2x^3)$$

6. Simplify.
   a) $(2x^4 - 3x^2 - 6) + (6x^4 - x^3 + 4x^2 + 5)$ c) $-7x(x^2 + x - 1) - 3x(2x^2 - 5x + 6)$

$$= 2x^4 - 3x^2 - 6 + 6x^4 - x^3 + 4x^2 + 5$$

$$= 8x^4 - x^3 + x^2 - 1$$

$$= -7x^3 - 7x^2 + 7x - 6x^3 + 15x^2 - 18x$$

$$= -13x^3 + 8x^2 - 11x$$

6. Simplify.

e) $-2x[5x - (2x - 7)] + 6x[3x - (1 + 2x)]$

$= -2x[5x - 2x + 7] + 6x[3x - 1 - 2x]$

$= -10x^2 + 4x^2 - 14x + 18x^2 - 6x - 12x^2$

$= -20x$

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6. Simplify.

g) $(x^2 + 5x - 3)^2$

	x^2	$5x$	-3
x^2	x^4	$5x^3$	$-3x^2$
$5x$	$5x^3$	$25x^2$	$-15x$
-3	$-3x^2$	$-15x$	9

$= x^4 + 10x^3 + 19x^2 - 30x + 9$

7. Factor.

a) $12m^2n^3 + 18m^3n^2$

$$= 6m^2n^2(2n + 3m)$$

e) $9x^2 - 6x + 1$

$$= (3x - 1)(3x - 1)$$
$$= (3x - 1)^2$$

c) $3x^2 + 24x + 45$

$$= 3(x^2 + 8x + 15)$$
$$= 3(x + 3)(x + 5)$$

9. Simplify. State any restrictions on the variables.

a) $\dfrac{10a^2b + 15bc^2}{-5b}$

$$b \neq 0$$

$$= -2a^2 - 3c^2$$

c) $\dfrac{xy - xyz}{xy}$

$$x \neq 0, y \neq 0$$

$$= \frac{xy}{xy} - \frac{xyz}{xy}$$

$$= 1 - z$$

10. Simplify. State any restrictions on the variables.

   a) $8xy^2 + 12x^2y - \dfrac{6x^3}{2xy}$

$$x \neq 0,\ y \neq 0$$

$$= 8xy^2 + 12x^2y - \frac{3x^2}{y}$$

$$= \frac{8xy^3}{y} + \frac{12x^2y^2}{y} - \frac{3x^2}{y}$$

$$= \frac{8xy^3 + 12x^2y^2 - 3x^2}{y}$$

10. Simplify. State any restrictions on the variables.

   c) $\dfrac{m+3}{m^2 + 10m + 21}$

$$= \frac{m+3}{(m+3)(m+7)} \qquad m \neq -3, -7$$

$$= \frac{(m+3)^{1}}{(m+3)(m+7)}$$

$$= \frac{1}{m+7}$$

10. Simplify. State any restrictions on the variables.

e) $\dfrac{3x^2 - 21x}{7x^2 - 28x + 21}$

$$= \frac{3x(x-7)}{7(x^2-4x+3)}$$

$$= \frac{3x(x-7)}{7(x-3)(x-1)} \qquad x \neq 3, 1$$

Can't simplify any further

11. If two rational functions have the same restrictions, are they equivalent? Explain and illustrate with an example.

No.

$$\frac{(x+1)(x+3)}{(x+1)} \quad \text{and} \quad \frac{x+4}{x+1}$$

Both have $x \neq -1$ as a restriction but they don't simplify to give the same expression.

12. Simplify. State any restrictions on the variables.

b) $\dfrac{10m^2}{3n} \times \dfrac{6mn}{20m^2}$

d) $\dfrac{5p}{8pq} \div \dfrac{3p}{12q}$

$n \neq 0, \; m \neq 0$

$= \dfrac{10m^2}{3n} \times \dfrac{6mn}{20m^2}$

$= \dfrac{60m^3 n}{60m^2 n}$

$= m$

$p \neq 0, \; q \neq 0$

$= \dfrac{5p}{8pq} \times \dfrac{12q}{3p}$

$= \dfrac{60pq}{24p^2 q}$

$= \dfrac{5}{2p}$

13. Simplify. State any restrictions on the variables.

b) $\dfrac{x^2 - 5x + 6}{x^2 - 1} \times \dfrac{x^2 - 4x - 5}{x^2 - 4} \div \dfrac{x - 5}{x^2 + 3x + 2}$

$x \neq \pm 1, \pm 2, 5$

$= \dfrac{(x-2)(x-3)}{(x+1)(x-1)} \times \dfrac{(x-5)(x+1)}{(x+2)(x-2)} \times \dfrac{(x+2)(x+1)}{(x-5)}$

$= \dfrac{(x-2)(x-3)}{(x+1)(x-1)} \times \dfrac{(x-5)(x+1)}{(x+2)(x-2)} \times \dfrac{(x+2)(x+1)}{(x-5)}$

$= \dfrac{(x-3)(x+1)}{x-1}$

13. Simplify. State any restrictions on the variables.

d) $\dfrac{x^2 - y^2}{4x^2 - y^2} \times \dfrac{4x^2 + 8xy + 3y^2}{x + y} \div \dfrac{2x + 3y}{2x - y}$ $y \neq \pm 2x, -x, -\dfrac{2}{3}x$

$$= \frac{(x+y)(x-y)}{(2x+y)(2x-y)} \times \frac{(2x+3y)(2x+y)}{(x+y)} \times \frac{2x-y}{2x+3y}$$

$$= \frac{\cancel{(x+y)}(x-y)}{\cancel{(2x+y)}\cancel{(2x-y)}} \times \frac{\cancel{(2x+3y)}\cancel{(2x+y)}}{\cancel{(x+y)}} \times \frac{\cancel{(2x-y)}}{\cancel{(2x+3y)}}$$

$$= x - y$$

14. Simplify. State any restrictions on the variables.

b) $\dfrac{5}{x + 1} - \dfrac{2}{x - 1}$ $x \neq \pm 1$

$$= \frac{5(x-1)}{(x+1)(x-1)} - \frac{2(x+1)}{(x-1)(x+1)}$$

$$= \frac{5x - 5}{(x+1)(x-1)} - \frac{2x + 2}{(x-1)(x+1)}$$

$$= \frac{5x - 5 - 2x - 2}{(x+1)(x-1)} \qquad = \frac{3x - 7}{(x+1)(x-1)}$$

14. Simplify. State any restrictions on the variables.

d) $\dfrac{1}{x^2 - 5x + 6} - \dfrac{1}{x^2 - 9}$ $x \neq 2, \pm 3$

$$= \frac{1}{(x-2)(x-3)} - \frac{1}{(x+3)(x-3)}$$

$$= \frac{1(x+3)}{(x-2)(x-3)(x+3)} - \frac{1(x-2)}{(x+3)(x-3)(x-2)}$$

$$= \frac{x+3}{(x-2)(x-3)(x+3)} - \frac{x-2}{(x+3)(x-3)(x-2)}$$

$$= \frac{x+3-x+2}{(x-2)(x+3)(x-3)} = \frac{5}{(x-2)(x+3)(x-3)}$$

15. Simplify and state any restrictions on the variables.

d) $\dfrac{2(x-2)^2}{x^2 + 6x + 5} \times \dfrac{3x + 15}{(2-x)^2}$ $x \neq -5, -1, 2$

$$= \frac{2\cancel{(x-2)}\cancel{(x-2)}}{\cancel{(x+5)}(x+1)} \times \frac{3\cancel{(x+5)}}{\underset{-1}{\cancel{(2-x)}}\,\underset{-1}{\cancel{(2-x)}}}$$

$$= \frac{6}{x+1}$$

15. Simplify and state any restrictions on the variables.

f) $\dfrac{2b-5}{b^2-2b-15} + \dfrac{3b}{b^2+b-30} \times \dfrac{b^2+8b+12}{b+3}$

$b \neq 5, -3, -6$

$= \dfrac{2b-5}{(b-5)(b+3)} + \dfrac{3b}{(b+6)(b-5)} \times \dfrac{(b+2)(b+6)}{b+3}$

Multiply BEFORE Adding (BEDMAS)

$= \dfrac{2b-5}{(b-5)(b+3)} + \dfrac{3b(b+2)}{(b-5)(b+3)}$

$= \dfrac{2b-5}{(b-5)(b+3)} + \dfrac{3b^2+6b}{(b-5)(b+3)}$

$= \dfrac{3b^2+8b-5}{(b-5)(b+3)}$

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16. Fred's final mark in an online course was determined entirely by two exams. The mid-term exam was out of x marks and was worth 25% of his final mark. The final exam was out of $2x$ marks and was worth 75% of his final mark. Fred scored 40 marks on the first exam and 60 marks on the second exam. Determine the value of x if Fred earned a final mark of 50% in the course.

$0.25\left(\dfrac{40}{x}\right) + 0.75\left(\dfrac{60}{2x}\right) = 0.50$

$= \dfrac{10}{x} + \dfrac{45}{2x} = 0.5$

$= \dfrac{20}{2x} + \dfrac{45}{2x} = 0.5$

$= \dfrac{65}{2x} = 0.5$

$65 = 0.5(2x)$

$65 = x$

14. A river flows at 2 km/h, and John takes 6 h to row 16 km up the river and back. How fast did he row?

	Distance	Speed	Time
Down	16	$x+2$	$\frac{16}{x+2}$
Back	16	$x-2$	$\frac{16}{x-2}$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \implies \text{Time} = \frac{\text{Dist}}{\text{Speed}}$$

$$T_{DOWN} + T_{BACK} = 6 \text{ hours}$$

$$\frac{16}{x+2} + \frac{16}{x-2} = 6$$

$$\frac{16(x-2)}{(x+2)(x-2)} + \frac{16(x+2)}{(x-2)(x+2)} = 6$$

$$\frac{16x-32}{(x+2)(x-2)} + \frac{16x+32}{(x-2)(x+2)} = 6$$

$$\frac{32x}{(x+2)(x-2)} = 6$$

$$32x = 6(x+2)(x-2)$$
$$32x = 6(x^2-4)$$
$$32x = 6x^2-24$$
$$0 = 6x^2-32x-24$$

Use quadratic formula

$$x = \frac{32 \pm \sqrt{(-32)^2 - 4(6)(-24)}}{2(6)}$$

$$x = \frac{32 \pm \sqrt{1600}}{12}$$

$$x = \frac{72}{12} = 6$$

$$x = \frac{-8}{12} = -\frac{2}{3}$$

Extraneous

$$\implies \text{Rowed at a speed of } 6 \text{ km/h}$$