

# Solutions

12. On the 42-km go-cart course at Sportsworld, Arshia drives 0.4 km/h faster than Sarah, but she has engine trouble part way around the course and has to stop to get the go-cart fixed. This stop costs Arshia one-half hour, and so she arrives 15 min after Sarah at the end of the course. How far did each girl drive and how long did each girl take to finish the course? Answer to one decimal place.

$\text{Speed} = \frac{\text{Dist}}{\text{Time}}$

	Dist	Speed	Time
Arshia	42	$x+0.4$	$\frac{42}{x+0.4}$
Sarah	42	$x$	$\frac{42}{x}$

Arshia stops for 30 mins, arrives 15 mins after Sarah

$$\frac{42}{x} - \frac{42}{x+0.4} = 0.25 \leftarrow 15 \text{ mins}$$

$$\frac{42(x+0.4)}{x(x+0.4)} - \frac{42(x)}{x(x+0.4)} = 0.25$$

$$\frac{42x + 16.8}{x(x+0.4)} - \frac{42x}{x(x+0.4)} = 0.25$$

$$\frac{16.8}{x(x+0.4)} = 0.25$$

$$16.8 = 0.25x(x+0.4)$$

$$16.8 = 0.25x^2 + 0.1x$$

$$0 = 0.25x^2 + 0.1x - 16.8$$

$$0 = 25x^2 + 10x - 1680$$

$$x = \frac{-10 \pm \sqrt{(10)^2 - 4(25)(-1680)}}{2(25)}$$

$$x = \frac{-10 \pm \sqrt{168100}}{50}$$

$$x = \frac{400}{50} \quad x = \frac{-420}{50}$$

$x = 8 \text{ km/h}$       $x = -8.4 \text{ km/h}$   
↑  
 Extraneous

Sarah = 8 km/h  
 Time =  $\frac{42}{8}$   
 =  $5\frac{1}{4}$  hours

Arshia = 8.4 km/h  
 Time =  $\frac{42}{8.4} + 0.5$   
 =  $5\frac{1}{2}$  hours

13. Rowing at 8 km/h, in still water, Rima and Bhanu take 16 h to row 39 km down a river and back. Find the speed of the current to two decimal places.

	Dist	Speed	Time
Downstream	39	$8+c$	$\frac{39}{8+c}$
Upstream	39	$8-c$	$\frac{39}{8-c}$

$$T_{\text{DOWN}} + T_{\text{UP}} = T_{\text{TOTAL}}$$

$$\frac{39}{8+c} + \frac{39}{8-c} = 16$$

$$\frac{39(8-c)}{(8+c)(8-c)} + \frac{39(8+c)}{(8-c)(8+c)} = 16$$

$$\frac{312-39c}{(8+c)(8-c)} + \frac{312+39c}{(8-c)(8+c)} = 16$$

$$\frac{624}{(8+c)(8-c)} = 16$$

$$624 = 16(8+c)(8-c)$$

$$\frac{624}{16} = \frac{16(64-c^2)}{16}$$

$$39 = 64 - c^2$$

$$c^2 - 25 = 0$$

$$\Rightarrow (c+5)(c-5) = 0$$

$$c = -5, c = 5$$

$\Rightarrow$  Speed of the current = 5 km/h

14. A river flows at 2 km/h, and John takes 6 h to row 16 km up the river and back. How fast did he row?

	Distance	Speed	Time
Down	16	$x+2$	$\frac{16}{x+2}$
Back	16	$x-2$	$\frac{16}{x-2}$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \Rightarrow \text{Time} = \frac{\text{Dist}}{\text{Speed}}$$

$$T_{\text{DOWN}} + T_{\text{BACK}} = 6 \text{ hours}$$

$$\frac{16}{x+2} + \frac{16}{x-2} = 6$$

$$\frac{16(x-2)}{(x+2)(x-2)} + \frac{16(x+2)}{(x-2)(x+2)} = 6$$

$$\frac{16x-32}{(x+2)(x-2)} + \frac{16x+32}{(x-2)(x+2)} = 6$$

$$\frac{32x}{(x+2)(x-2)} = 6$$

$$32x = 6(x+2)(x-2)$$

$$32x = 6(x^2-4)$$

$$32x = 6x^2 - 24$$

$$0 = 6x^2 - 32x - 24$$

Use quadratic formula

$$x = \frac{32 \pm \sqrt{(-32)^2 - 4(6)(-24)}}{2(6)}$$

$$\rightarrow x = \frac{32 \pm \sqrt{1600}}{12}$$

$$x = \frac{72}{12} = 6$$

$$x = \frac{-8}{12} = -\frac{2}{3}$$

Extraneous

$\Rightarrow$  Rowed at a speed of 6 km/h

15. Application: Jaime bought a case of concert T-shirts for \$450. She kept two for herself and sold the rest for \$560, making a profit of \$10 on each shirt. How many shirts were in the case?

Let  $n$  = # of t-shirts  
 $t$  = cost of one t-shirt

$$\Rightarrow nt = 450 \qquad n = \frac{450}{t}$$

$$(n-2)(t+10) = 560$$

$$\left(\frac{450}{t} - 2\right)(t+10) = 560$$

$$450 + 10\left(\frac{450}{t}\right) - 2t - 20 = 560$$

$$\frac{4500}{t} - 2t - 130 = 0$$

$$4500 - 2t^2 - 130t = 0$$

$$0 = 2t^2 + 130t - 4500$$

$$0 = t^2 + 65t - 2250$$

$$t = \frac{-65 \pm \sqrt{(65)^2 - 4(1)(-2250)}}{2(1)}$$

$$t = \frac{-65 \pm \sqrt{13225}}{2}$$

$$t = \frac{50}{2} \text{ or } \frac{-180}{2}$$

$$t = 25 \text{ or } -90$$

↑  
Extraneous

Using  $n = \frac{450}{t}$   
 $n = \frac{450}{25}$   
 $n = 18$

16. Stuart agrees to a house-painting job for \$900. He takes 4 days longer than expected, and he has earned \$18.75 less per day than expected. In how many days did he expect to complete the house?

Let  $n$  = # of days  
 $p$  = pay per day

$$\Rightarrow np = 900 \qquad n = \frac{900}{p}$$

$$(n+4)(p-18.75) = 900$$

$$\left(\frac{900}{p} + 4\right)(p-18.75) = 900$$

$$900 - 18.75\left(\frac{900}{p}\right) + 4p - 75 = 900$$

$$-\frac{16875}{p} + 4p - 75 = 0$$

$$-16875 + 4p^2 - 75p = 0$$

$$4p^2 - 75p - 16875 = 0$$

$$p = \frac{75 \pm \sqrt{(-75)^2 - 4(4)(-16875)}}{2(4)}$$

$$p = \frac{75 \pm \sqrt{275625}}{8}$$

$$p = \frac{600}{8} \text{ or } \frac{-450}{8}$$

$$p = 75 \text{ or } -56.25$$

↑  
Extraneous

Using  $n = \frac{900}{p}$   
 $n = \frac{900}{75}$   
 $n = 12 \text{ days}$

Let  $n = \#$  of students  
 $p =$  price of one lunch

$$\Rightarrow np = 239.25$$

$$\text{and } (n-4)(p+1) = 239.25 \rightarrow p = \frac{239.25}{n}$$

$$(n-4)\left(\frac{239.25}{n} + 1\right) = 239.25$$

$$\cancel{239.25} + n - \frac{957}{n} - 4 = \cancel{239.25}$$

$$n^2 - 957 - 4n = 0$$

$$n^2 - 4n - 957 = 0$$

$$n = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-957)}}{2(1)} \quad n = \frac{66}{2} \text{ or } \frac{-58}{2}$$

$$n = \frac{4 \pm \sqrt{3844}}{2}$$

$$n = 33 \text{ or } -29$$

Students  
 Extraneous

17. A grade 11 class, on a field trip to Montreal, had lunch in a restaurant. The bill came to \$239.25. Four students had birthdays that day, and it was agreed that these four should not have to pay for lunch. The other students had to pay \$1 more than if all the students had paid. How many students had lunch?