

Solving Rational Problems

Lesson objectives

- I know how to create an equation to represent a problem
- I know how to solve speed/distance/time problems

1.1

Lesson objectives

Teachers' notes

Lesson notes

Handout 3.80 #s 12, 14, 15 & 16

Example:

A Grade 11 class, on a field trip to Montreal, had lunch in a restaurant. The bill came to \$239.25. Four students had birthdays that day, and it was agreed that these four should not have to pay for lunch. The other students had to pay \$1 more than if all the students had paid. How many students had lunch?

Let n = # of students and p = price of one lunch

$$n(p) = 239.25 \quad (1) \quad (\text{cost if everyone paid})$$

$$(n - 4)(p + 1) = 239.25 \quad (2) \quad (\text{cost if birthday students don't pay})$$

From (1) we get $p = 239.25/n$

Sub this into (2) and we get $(n - 4)(239.25/n + 1) = 239.25$

If we FOIL we get

$$239.25 + n - 957/n - 4 = 239.25$$

Simplify:

$$(-239.25) \quad n - 957/n - 4 = 0$$

$$(\text{multiply by } n) \quad n^2 - 957 - 4n = 0$$

We could try to factor (and it should, as we are solving for a number of people) but it is likely easier to use the quadratic formula to solve for n

$$n = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-957)}}{2(1)}$$

$$n = \frac{4 \pm \sqrt{16 - (-3828)}}{2}$$

$$n = \frac{4 \pm \sqrt{3844}}{2} \quad n = 33 \text{ or } -29 \leftarrow \text{Extraneous}$$

There were 33 students that had lunch

Example: Rowing at 8 km/h, in still water, Rima and Bhanu take 16 h to row 39 km down a river and back. Find the speed of the current to two decimal places.

When rowing with the current their speed will increase, when rowing against the current their speed will decrease.

Let c = speed of the current

Time = Distance / Speed

	Distance	Speed	Time
Down river	39 km	$8 + c$	$39/(8 + c)$
Up river	39 km	$8 - c$	$39/(8 - c)$

Total time taken = time down river + time up river = 16 hours

$$\rightarrow \frac{39}{8+c} + \frac{39}{8-c} = 16$$

$$\frac{39(8-c)}{(8+c)(8-c)} + \frac{39(8+c)}{(8-c)(8+c)} = 16$$

$$39(8-c) + 39(8+c) = 16(8+c)(8-c)$$

$$312 - 39c + 312 + 39c = 16(64 - c^2)$$

$$624 = 1024 - 16c^2$$

$$-400 = -16c^2$$

$$\frac{-400}{-16} = \frac{-16c^2}{-16}$$

$$25 = c^2$$

$$\sqrt{25} = c$$

Speed of the current = 5.00 km/h to 2 decimal places (-5.00 km/h is extraneous)

Example

Ero and Jamal set off at the same time on a 30-km walk for charity. Ero, who has trained all year for this event, walks 1.4 km/h faster than Jamal, but sees a friend on the route and stops to talk for 20 min. Even with this delay, Ero finishes the walk 2 h ahead of Jamal. How fast was each person walking, and how long did it take for each person to finish the walk?

	Speed	Distance	Time
Ero	$x + 1.4$	30	$\frac{30}{x+1.4}$
Jamal	x	30	$\frac{30}{x}$

$$T_{\text{Jamal}} - T_{\text{Ero}} = 2 \text{ hours } 20 \text{ mins}$$

$$\frac{30}{x} - \frac{30}{x+1.4} = \frac{7}{3} \quad \left[2\frac{1}{3} \text{ hrs}\right]$$

$$\frac{30(x+1.4)}{x(x+1.4)} - \frac{30(x)}{x(x+1.4)} = \frac{7}{3}$$

$$\frac{30x + 42}{x(x+1.4)} - \frac{30x}{x(x+1.4)} = \frac{7}{3}$$

$$\frac{42}{x(x+1.4)} = \frac{7}{3}$$

$$42(3) = 7x(x+1.4)$$

$$126 = 7x^2 + 9.8x$$

$$0 = 7x^2 + 9.8x - 126$$

$$x = \frac{-9.8 \pm \sqrt{(9.8)^2 - 4(7)(-126)}}{2(7)}$$

$$x = \frac{-9.8 \pm \sqrt{3624.04}}{14}$$

$$x = \frac{-9.8 \pm 60.2}{14}$$

$$x = 3.6 \text{ km/h}$$

$$x = -5 \text{ km/h}$$

Extraneous

Jamal speed
= 3.6 km/h

Jamal time
= $\frac{30}{3.6} = 8.\bar{3}$
= 8 hours 20 mins

Ero speed
= 3.6 + 1.4
= 5 km/h

Ero time
= $\frac{30}{5} = 6$ hours
= 6 + 20 mins
= 6 hours 20 mins