

Solutions

11. Solve for x.

(a) $\frac{x-2}{x} + \frac{4}{5x} = -\frac{1}{5}$ (b) $\frac{2x+3}{x-1} - \frac{3}{x} = 2$

$x \neq 0$ $x \neq 1, x \neq 0$

$$= \frac{5x(x-2)}{5x(x)} + \frac{4(x)}{5x(x)} = -\frac{1}{5}$$

$$= \frac{5x^2 - 10x}{5x^2} + \frac{4x}{5x^2} = -\frac{1}{5}$$

$$= \frac{5x^2 - 10x + 4x}{5x^2} = -\frac{1}{5}$$

$$= \frac{5x^2 - 6x}{5x^2} = -\frac{1}{5}$$

$$= 5(5x^2 - 6x) = -1(5x^2)$$

$$= 25x^2 - 30x = -5x^2$$

$$= 30x^2 - 30x = 0$$

$$= 30x(x-1) = 0$$

$$x = 0, x = 1$$

Not valid
 $\Rightarrow x = 1$

$$= \frac{x(2x+3)}{x(x-1)} - \frac{3(x-1)}{x(x-1)} = 2$$

$$= \frac{2x^2 + 3x}{x(x-1)} - \frac{3x-3}{x(x-1)} = 2$$

$$= \frac{2x^2 + 3x - 3x + 3}{x(x-1)} = 2$$

$$= \frac{2x^2 + 3}{x(x-1)} = 2$$

$$= 2x^2 + 3 = 2(x)(x-1)$$

$$= 2x^2 + 3 = 2x(x-1)$$

$$= 2x^2 + 3 = 2x^2 - 2x$$

$$= \frac{3}{-2} = \frac{-2x}{-2}$$

$$x = -\frac{3}{2}$$

11. Solve for x.

(c) $\frac{1}{x} = \frac{2}{x+1} + \frac{1}{1-x}$ (d) $\frac{3x}{x^2-1} = \frac{x}{x+1} - 4$

$x \neq 0, x \neq -1, x \neq 1$

$$\frac{1}{x} = \frac{2(1-x)}{(x+1)(1-x)} + \frac{1(x+1)}{(1-x)(x+1)}$$

$$\frac{1}{x} = \frac{2-2x}{(x+1)(1-x)} + \frac{x+1}{(1-x)(x+1)}$$

$$\frac{1}{x} = \frac{2-2x+x+1}{(x+1)(1-x)}$$

$$\frac{1}{x} = \frac{3-x}{(x+1)(1-x)}$$

$$1(x+1)(1-x) = x(3-x)$$

$$x-x^2+1-x = 3x-x^2$$

$$1-x^2 = 3x-x^2$$

$$\frac{1}{3} = \frac{3x}{3}$$

$$\Rightarrow x = \frac{1}{3}$$

(d)

$$\frac{3x}{(x+1)(x-1)} = \frac{x}{x+1} - 4$$

$x \neq -1, 1$

$$\frac{3x}{(x+1)(x-1)} = \frac{x(x-1)}{(x+1)(x-1)} - 4$$

$$\frac{3x}{(x+1)(x-1)} = \frac{x^2-x}{(x+1)(x-1)} - 4$$

$$\frac{3x}{(x+1)(x-1)} - \frac{x^2-x}{(x+1)(x-1)} = -4$$

$$\frac{3x-x^2+x}{(x+1)(x-1)} = -4$$

$$\frac{-x^2+4x}{(x+1)(x-1)} = -4$$

$$-x^2+4x = -4(x+1)(x-1)$$

$$-x^2+4x = -4(x^2-1)$$

$$-x^2+4x = -4x^2+4$$

$$3x^2+4x-4 = 0$$

$$(3x-2)(x+2) = 0$$

$$\Rightarrow x = \frac{2}{3} \quad x = -2$$

11. Solve for x.

(e) $\frac{15}{x^2-1} = \frac{4}{x-1} - \frac{3}{x+1}$

$x \neq 1, -1$

$$\frac{15}{(x+1)(x-1)} = \frac{4}{x-1} - \frac{3}{x+1}$$

$$\frac{15}{(x+1)(x-1)} = \frac{4(x+1)}{(x-1)(x+1)} - \frac{3(x-1)}{(x+1)(x-1)}$$

$$\Rightarrow 15 = 4(x+1) - 3(x-1)$$

$$15 = 4x + 4 - 3x + 3$$

$$15 = x + 7$$

$$8 = x$$

11. Solve for x.

$$(f) \frac{x}{x-2} + 2 = \frac{5x}{x+2} + \frac{3x+1}{x^2-4}$$

$$x \neq -2, 2$$

$$\frac{x}{x-2} + 2 = \frac{5x}{x+2} + \frac{3x+1}{(x+2)(x-2)}$$

$$\frac{x(x+2)}{(x-2)(x+2)} + \frac{2(x-2)(x+2)}{(x-2)(x+2)} = \frac{5x(x-2)}{(x+2)(x-2)} + \frac{3x+1}{(x+2)(x-2)}$$

$$\Rightarrow x(x+2) + 2(x-2)(x+2) = 5x(x-2) + 3x+1$$

$$x^2 + 2x + 2(x^2 - 4) = 5x^2 - 10x + 3x + 1$$

$$x^2 + 2x + 2x^2 - 8 = 5x^2 - 7x + 1$$

$$3x^2 + 2x - 8 = 5x^2 - 7x + 1$$

$$0 = 2x^2 - 9x + 9$$

$$0 = (2x-3)(x-3)$$

$$x = \frac{3}{2}, 3$$