Combinations

Extra Practice

MHR Page 136 #s 1 - 11 & 15 - 17

Solutions

1. How many ways are there to select four people from a group of nine people, without regard to order?

A 36 B 262 144

C 126 D 3024

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$

$$\Rightarrow {}_9C_4 = \frac{9!}{(9-4)!\,4!}$$

$$= \frac{9!}{5!\,4!}$$

$$= 126 \Rightarrow C$$

2. What is the total number of subsets of a set of 10 elements?

A 1024 B 1023 C 100 D 20

Indirect method

Can't have a subset with no elements

$$\Rightarrow 2^{10} - 1$$

include/not include

# of elements

subtract the null set

$$= 1024 - 1$$

$$= 1023$$

$$\Rightarrow B$$

3. Using Pascal's method, what is ${}_7C_3 + {}_7C_4$?

A ${}_8C_3$ **B** ${}_8C_4$ **C** ${}_8C_5$ **D** ${}_7C_5$

$${}_nC_r + {}_nC_{r+1} = {}_{n+1}C_{r+1}$$

$$\Rightarrow {}_7C_3 + {}_7C_4$$

$$= {}_8C_4 \Rightarrow B$$

4. What is the number of arrangements of three red and two green blocks?

A $\frac{5!}{3!2!}$ **B** $3! \times 2!$

C $5!$ **D** $\frac{6!}{3!2!}$

$$n(A) = \frac{n!}{p!\,q!}$$

$$\Rightarrow \frac{5!}{3!\,2!}$$

Total # of blocks

# of red blocks

# of green blocks

$\Rightarrow A$

5. In how many ways could a 6-member committee be formed from a 16-member club, if the president and secretary must be on the committee?

Select President Secretary first.
We are left needing 4 more members from 14 that remain.

$\Rightarrow {}_{14}C_4 = 1001$ ways

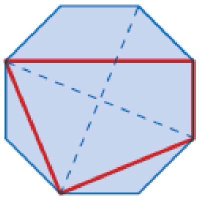
6. You found seven library books that you would like to take out, but the maximum is four. In how many ways could you select the four books?

${}_7C_4 = 35$ ways

seven books

can only take four of them

7. How many quadrilaterals can be formed from the vertices of an octagon?



8 vertices on an octagon
4 vertices on a quadrilateral

$\Longrightarrow {}_8C_4 = 70$ quadrilaterals

8. How many permutations are there of the letters in the word RELATIONS, if the vowels must be in alphabetical order?

Treat the vowels as identical objects as they must stay in one order only.
9 letters, 4 vowels

$\Longrightarrow n(A) = \dfrac{9!}{4!}$

$= 15,120$ permutations

9. **a)** How are ${}_8C_3$ and ${}_8P_3$ related?

b) Explain this relationship. Include an example to support your explanation.

a) $_8C_3 = \frac{_8P_3}{3!}$

b) Both represent choosing 3 objects from 8. Order is not important for $_8C_3$ but is for $_8P_3$.

Example: Choosing 6 players from 9 to play hockey. $\Rightarrow {}_9C_6 = 84$

Choosing 6 players from 9, needing 1 Goalie, 1 LD, 1 RD, 1 LW, 1 C, 1 RW

$\Rightarrow {}_9P_6 = 60{,}480$

10. Two balls are selected from a bag with five white and nine black balls. What is the probability that both balls are black?

5 white
9 black
14 in total

$n(A) = {}_{14}C_2$

$n(2\text{ black}) = {}_9C_2 \times {}_5C_0$

2 blacks from 9 — AND — 0 whites from 5

$P(2\text{ black}) = \frac{_9C_2 \times {}_5C_0}{_{14}C_2}$

$= \frac{36 \times 1}{91}$

$= \frac{36}{91}$ $(\approx 0.3956\ldots)$

11. What is the coefficient of p^4q^6 in the expansion of $(p+q)^{10}$?

1ST term has p^{10}
2ND term has p^9
3RD term has p^8
⋮
7TH term has p^4
Need to find row 10, 7th term

$\Rightarrow {}_{10}C_6 = 210$

<u>Recall</u> 1ST term would be ${}_{10}C_0$

15. Mario orders a pizza with 3 toppings, chosen from 15 available toppings.

a) In how many ways could mushrooms or olives be included in his toppings?

b) Would the result in part a) be greater or less if he orders 4 toppings? Explain.

a) $n(\text{arrangements}) = {}_{15}C_3$
$n(\text{No olives or mushrooms}) = {}_{13}C_3$

can't choose olives or mushrooms

$\Rightarrow n(\text{include O or M}) = {}_{15}C_3 - {}_{13}C_3$
$= 455 - 286$
$= 169$ ways

b) For 4 toppings
$n(A) = {}_{15}C_4$
$n(\text{no O or M}) = {}_{13}C_4$
$\Rightarrow n(\text{include O or M}) = {}_{15}C_4 - {}_{13}C_4$
$= 1365 - 715$
$= 650$ ways
$\Rightarrow$ More options if you can have 4 toppings.

16. A package of 50 computer chips contains 45 that are perfect and 5 that are defective. If 2 chips are selected at random, what is the probability that

a) neither is defective?

b) both are defective?

c) only one is defective?

$n(A) = {}_{50}C_2$

a) $n(\text{neither defective}) = {}_{45}C_2 \times {}_5C_0$

$\Rightarrow P(\text{neither defective}) = \dfrac{{}_{45}C_2 \times {}_5C_0}{{}_{50}C_2} = \dfrac{990 \times 1}{1225} = \dfrac{198}{245}$ $(\approx 0.80816\ldots)$

b) $n(\text{both defective}) = {}_5C_2 \times {}_{45}C_0$

$\Rightarrow P(\text{both defective}) = \dfrac{{}_5C_2 \times {}_{45}C_0}{{}_{50}C_2} = \dfrac{10 \times 1}{1225} = \dfrac{2}{245}$ $(\approx 0.00816\ldots)$

c) $n(\text{only one defective}) = {}_{45}C_1 \times {}_5C_1$

$\Rightarrow P(\text{only one defective}) = \dfrac{{}_{45}C_1 \times {}_5C_1}{{}_{50}C_2} = \dfrac{45 \times 5}{1225} = \dfrac{225}{1225} = \dfrac{9}{49}$ $(\approx 0.18367\ldots)$

17. The tens, jacks, queens, kings, and aces are removed from a standard deck of cards. From these cards, four are chosen. What is the probability that

a) all are queens?

b) all are red?

c) two are face cards?

d) there is at least one ace?

e) there are at least one ace and one king?

$n(A) = {}_{20}C_4$ (choose 4 from 20)

a) $n(\text{all queens}) = {}_4C_4 \times {}_{16}C_0$

$\Rightarrow P(\text{all queens}) = \dfrac{{}_4C_4 \times {}_{16}C_0}{{}_{20}C_4} = \dfrac{1 \times 1}{4845} = \dfrac{1}{4845}$ $(\approx 0.0002\ldots)$

b) $n(\text{all red}) = {}_{10}C_4 \times {}_{10}C_0$

$\Rightarrow P(\text{all red}) = \dfrac{{}_{10}C_4 \times {}_{10}C_0}{{}_{20}C_4} = \dfrac{210 \times 1}{4845} = \dfrac{210}{4845} = \dfrac{14}{323}$ $(\approx 0.04334\ldots)$

17. The tens, jacks, queens, kings, and aces are removed from a standard deck of cards. From these cards, four are chosen. What is the probability that

a) all are queens?

b) all are red?

c) two are face cards?

d) there is at least one ace?

e) there are at least one ace and one king?

$n(A) = {}_{20}C_4$ (choose 4 from 20)

c) $n(\text{2 face cards}) = {}_{12}C_2 \times {}_8C_2$

$$\Rightarrow P(\text{2 face cards}) = \frac{{}_{12}C_2 \times {}_8C_2}{{}_{20}C_4} = \frac{66 \times 28}{4845} = \frac{1848}{4845} = \frac{616}{1615} \quad (\approx 0.3814\ldots)$$

d) Use indirect method

$n(\text{no aces}) = {}_4C_0 \times {}_{16}C_4$

$$\Rightarrow P(\text{at least one ace}) = 1 - P(\text{no ace}) = 1 - \frac{{}_4C_0 \times {}_{16}C_4}{{}_{20}C_4} = 1 - \frac{1 \times 1820}{4845} = 1 - \frac{1820}{4845} = 1 - \frac{364}{969} = \frac{605}{969} \quad (\approx 0.624355\ldots)$$

17. The tens, jacks, queens, kings, and aces are removed from a standard deck of cards. From these cards, four are chosen. What is the probability that

a) all are queens?

b) all are red?

c) two are face cards?

d) there is at least one ace?

e) there are at least one ace and one king?

$n(A) = {}_{20}C_4$ (choose 4 from 20)

e) Use indirect method

$n(\text{no ace}) = {}_4C_0 \times {}_{16}C_4$

$n(\text{no king}) = {}_4C_0 \times {}_{16}C_4$

$n(\text{no ace and king}) = {}_{12}C_4$

already counted so add them back in

$\Rightarrow P(\text{at least one ace and one king})$

$$= 1 - P(\text{no ace}) - P(\text{no king}) + P(\text{no ace and king})$$

$$= 1 - \frac{{}_4C_0 \times {}_{16}C_4}{{}_{20}C_4} - \frac{{}_4C_0 \times {}_{16}C_4}{{}_{20}C_4} + \frac{{}_{12}C_4}{{}_{20}C_4}$$

$$= 1 - \frac{1 \times 1820}{4845} - \frac{1 \times 1820}{4845} + \frac{495}{4845}$$

$$= 1 - \frac{3145}{4845}$$

$$= \frac{1700}{4845}$$

$$= \frac{20}{57} \quad (\approx 0.351\ldots)$$