Completing the Square Algebraically

Lesson objectives

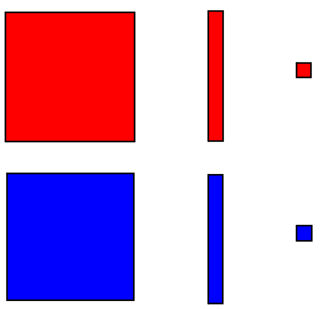
- I know how to complete the square with algebra tiles
- I understand and can carry out the algebraic process of completing the square

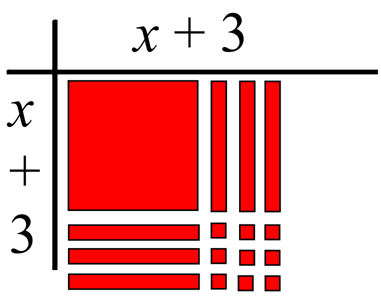
1.1

Lesson objectives | Teachers' notes | Lesson notes

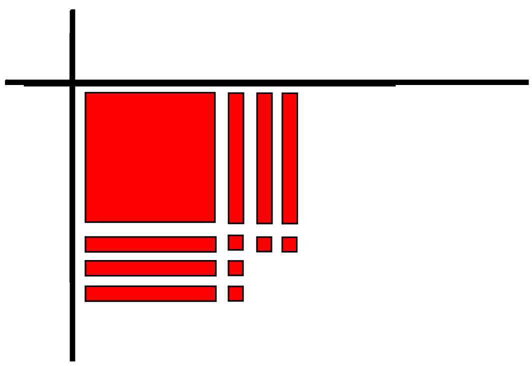
MHR Page 270 #s 2 - 5, 7ace, 8bd, 12, 15, 19 & 23

Show that the following is a perfect square: $x^2 + 6x + 9$





Now consider the quadratic expression: $x^2 + 6x + 5$



How is this related to a perfect square?

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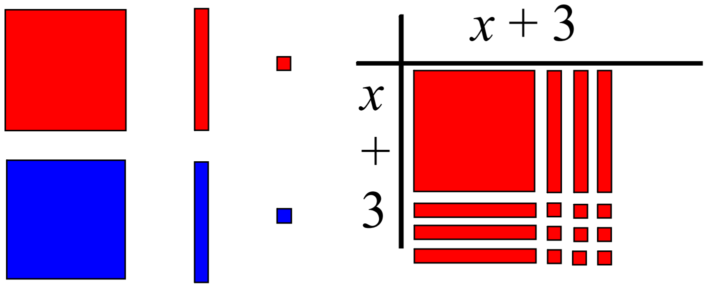

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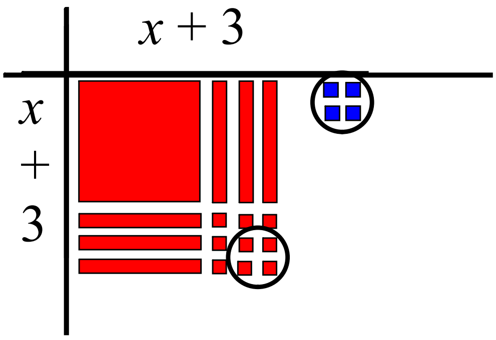
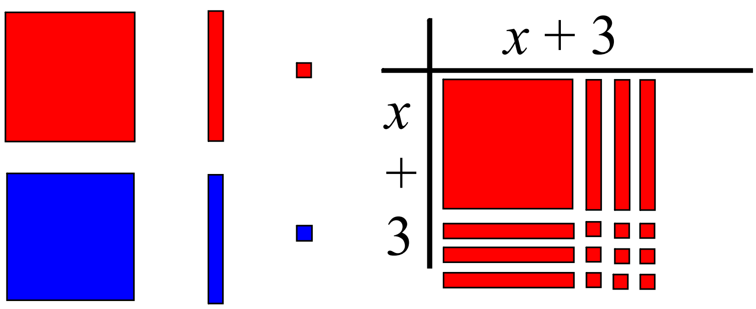


How is this related to a perfect square?

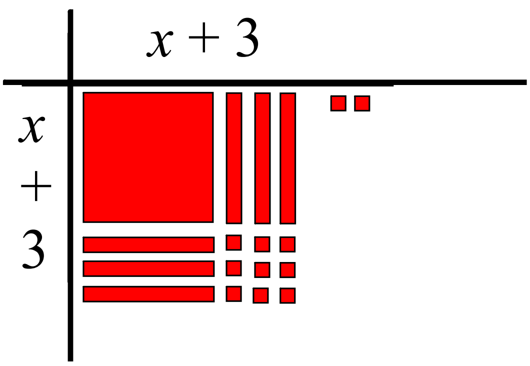
$$x^2 + 6x + 5 = (x + 3)^2 - 4$$

Use zero pairs to fill in the missing space. If we add four red "ones" we need to add four blue "ones" to keep it balanced.

Show that the following is a perfect square: $x^2 + 6x + 9$



Now consider the quadratic expression: $x^2 + 6x + 11$



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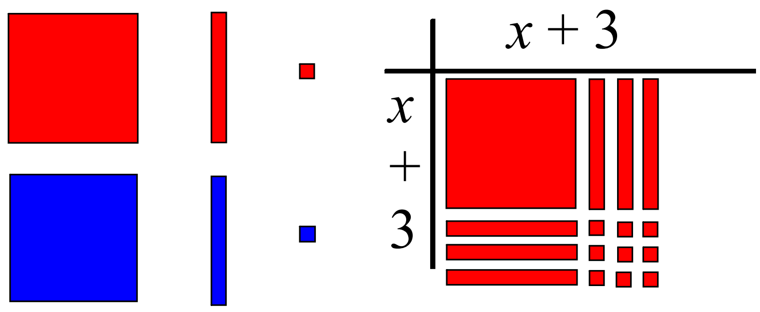

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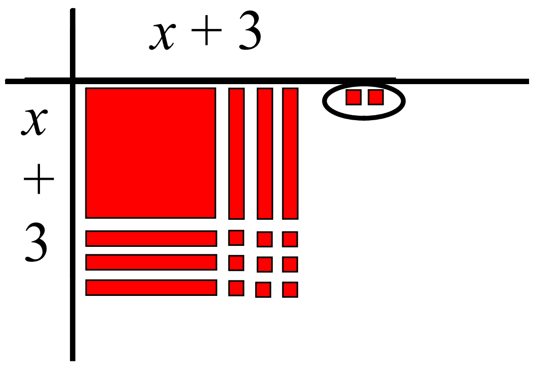

How is this related to a perfect square?

$$x^2 + 6x + 11 = (x + 3)^2 + 2$$

Steps for Completing the Square

1. Common factor the value of "a" from the x^2 and x terms.

2. Decompose the x term by splitting it in half.

3. Take the new coefficient of x and square it. Add and subtract this value inside the bracket.

4. Take the subtracted value inside the bracket, multiply it by the value of "a" and remove it from inside the bracket to outside the bracket.

5. Inside the bracket you now have a perfect square trinomial - factor.

6. Simplify the constant outside the bracket.

Example - Convert to Vertex Form

$y = x^2 + 12x + 12$

$y = 1(x^2 + 12x) + 12$

$y = (x^2 + 12x + (6)^2 - (6)^2) + 12$

$y = (x^2 + 12x + (6)^2) - 1(6)^2 + 12$

$y = (x + 6)^2 - 1(36) + 12$

$y = (x + 6)^2 - (36) + 12$

$y = (x + 6)^2 - 24$

1. Take a factor of "a" (1) from the first two terms.

2. & 3. Take half of the coefficient of x, square it, add it, square it, subtract it.

4. The term that is removed from the bracket needs to be multiplied by the "a" value (1) that was removed in step one.

The "a" value is positive, so we will have a minimum value (happy face) of -24.

Example - Convert to Vertex Form

$y = 2x^2 + 4x - 3$

$y = 2(x^2 + 2x) - 3$

$y = 2(x^2 + 2x + (1)^2 - (1)^2) - 3$

$y = 2(x^2 + 2x + (1)^2) - 2(1)^2 - 3$

$y = 2(x + 1)^2 - 2(1) - 3$

$y = 2(x + 1)^2 - (2) - 3$

$y = 2(x + 1)^2 - 5$

1. Take a factor of "a" (2) from the first two terms.

2. & 3. Take half of the coefficient of x, square it, add it, square it, subtract it.

4. The term that is removed from the bracket needs to be multiplied by the "a" value (2) that was removed in step one.

The "a" value is positive, so we will have a minimum value (happy face) of -5.

Example - Convert to Vertex Form

$y = 4x^2 + 24x + 10$

$y = 4(x^2 + 6x) + 10$

$y = 4(x^2 + 6x + (3)^2 - (3)^2) + 10$

$y = 4(x^2 + 6x + (3)^2) - 4(3)^2 + 10$

$y = 4(x + 3)^2 - 4(9) + 10$

$y = 4(x + 3)^2 - (36) + 10$

$y = 4(x + 3)^2 - 26$

1. Take a factor of "a" (4) from the first two terms.

2. & 3. Take half of the coefficient of x, square it, add it, square it, subtract it.

4. The term that is removed from the bracket needs to be multiplied by the "a" value (4) that was removed in step one.

The "a" value is positive, so we will have a minimum value (happy face) of -26.

Example - Convert to Vertex Form

$y = -2x^2 - 10x + 3$

$y = -2(x^2 + 5x) + 3$

$y = -2(x^2 + 5x + (2.5)^2 - (2.5)^2) + 3$

$y = -2(x^2 + 5x + (2.5)^2) - -2(2.5)^2 + 3$

$y = -2(x + 2.5)^2 - -2(6.25) + 3$

$y = -2(x + 2.5)^2 - (-12.5) + 3$

$y = -2(x + 2.5)^2 + 12.5 + 3$

$y = -2(x + 2.5)^2 + 15.5$

1. Take a factor of "a" (-2) from the first two terms.

2. & 3. Take half of the coefficient of x, square it, add it, square it, subtract it.

4. The term that is removed from the bracket needs to be multiplied by the "a" value (-2) that was removed in step one.

The "a" value is negative, so we will have a maximum value (sad face) of 15.5.

Example - Convert to Vertex Form

$y = -3x^2 + 24x - 10$

$y = -3(x^2 - 8x) - 10$

$y = -3(x^2 - 8x + (-4)^2 - (-4)^2) - 10$

$y = -3(x^2 - 8x + (-4)^2) - -3(-4)^2 - 10$

$y = -3(x - 4)^2 - -3(16) - 10$

$y = -3(x - 4)^2 - (-48) - 10$

$y = -3(x - 4)^2 + 48 - 10$

$y = -3(x - 4)^2 + 38$

1. Take a factor of "a" (-3) from the first two terms.

2. & 3. Take half of the coefficient of x, square it, add it, square it, subtract it.

4. The term that is removed from the bracket needs to be multiplied by the "a" value (-3) that was removed in step one.

The "a" value is negative, so we will have a maximum value (sad face) of 38.