Factor a Perfect Square Trinomial and a Difference of Squares

Lesson objectives

- I know how to factor the difference of squares
- I know how to factor a perfect square trinomial

1.1

Lesson objectives | Teachers' notes | Lesson notes

MHR Page 253 #s 1bdfh, 2aceg, 3 - 5, 6acdgh, 7, 8ace, 10, 11, 17 & 20acf

Warm up

Convert the following from standard form to factored form:

$y = 5x^2 - 11x + 2$

$(a)(c) = (5)(2) = 10$

Factors

-1×-10

-2×-5

$-1 + -10 = -11$

$-1 \times -10 = 10$

	$5x$	-1
x	$5x^2$	$-x$
-2	$-10x$	2

$\Rightarrow y = (x-2)(5x-1)$

$y = -3x^2 + 11x + 4$

Common factor: -1

$\Rightarrow y = -(3x^2 - 11x - 4)$

$(a)(c) = (3)(-4) = -12$

Factors

1×-12

2×-6

3×-4

$1 + -12 = -11$

$1 \times -12 = -12$

$y = -(3x^2 + x - 12x - 4)$

$y = -[x(3x+1) - 4(3x+1)]$

$y = -(3x+1)(x-4)$

Anytime we are factoring, our first step needs to be to look for a common factor.

This step must be done in order to fully factor any polynomial.

Note: The chart WILL NOT work if a common factor is left in!

$y = 4x^2 + 18x + 8$

Common factor : 2

$y = 2(2x^2 + 9x + 4)$

$(a)(c) = (2)(4) = 8$

$1 + 8 = 9$

$1 \times 8 = 8$

$y = 2[2x^2 + x + 8x + 4]$

$y = 2[x(2x+1) + 4(2x+1)]$

$y = 2(2x+1)(x+4)$

$y = 2x^2 + 10x + 12$

Common factor : 2

$y = 2(x^2 + 5x + 6)$

$(a)(c) = (1)(6) = 6$

$2 + 3 = 5$

$2 \times 3 = 6$

	x	2
x	x^2	$2x$
3	$3x$	6

$y = 2(x+3)(x+2)$

The Difference of Squares

A difference of squares binomial is when we have a quadratic that only has two terms (ax^2 and c) which are subtracted.
In this case the value of b is 0.

Factor the following:

a) $x^2 - 9$ $= x^2 + 0x - 9$
$= x^2 + 3x - 3x - 9$
$= x(x+3) - 3(x+3)$
$= (x+3)(x-3)$

Factors
1×-9
3×-3
9×-1

b) $x^2 - 100$ $= x^2 + 0x - 100$
$= x^2 + 10x - 10x - 100$
$= x(x+10) - 10(x+10)$
$= (x+10)(x-10)$

Factors
1×-100
2×-50
4×-25
5×-20
10×-10

c) $4x^2 - 36$ $= 4(x^2 - 9)$
$= 4(x^2 + 0x - 9)$
$= 4(x^2 + 3x - 3x - 9)$
$= 4[x(x+3) - 3(x+3)]$
$= 4(x+3)(x-3)$

Factors
1×-9
3×-3
9×-1

d) $16 - 9x^2$ $= -1(9x^2 - 16)$
$= -1(9x^2 + 0x - 16)$
$= -1(9x^2 + 12x - 12x - 16)$
$= -1[3x(3x+4) - 4(3x+4)]$
$= -(3x+4)(3x-4)$

Factors
1×-144
2×-72
3×-48
4×-36
6×-24
8×-18
9×-16
12×-12

Can always be factored using the chart ordecomposition.

Short cut is:

$$(\sqrt{ax^2} + \sqrt{c})(\sqrt{ax^2} - \sqrt{c})$$

Examples:

$25x^2 - 49$

$\sqrt{25x^2} = 5x$

$\sqrt{49} = 7$

$\Rightarrow (5x+7)(5x-7)$

$16 - 9x^2$

$\sqrt{16} = 4$

$\sqrt{9x^2} = 3x$

$\Rightarrow (4+3x)(4-3x)$

Perfect Square Trinomials

These are where ax^2 and c are both perfect squares and when you find two numbers that multiply to ac and add to b, those numbers will be the same.

Factor:

a) $x^2 + 6x + 9$

$= x^2 + 3x + 3x + 9$

$= x(x+3) + 3(x+3)$

$= (x+3)(x+3)$

Factors
1 x 9
3 x 3

b) $4x^2 - 12x + 9$

$= 4x^2 - 6x - 6x + 9$

$= 2x(2x-3) - 3(2x-3)$

$= (2x-3)(2x-3)$

$= (2x-3)^2$

Factors
-1 x -36
-2 x -18
-3 x -12
-4 x -9
-6 x -6

c) $9x^2 + 6x + 1$

$= 9x^2 + 3x + 3x + 1$

$= 3x(3x+1) + 1(3x+1)$

$= (3x+1)(3x+1)$

$= (3x+1)^2$

Factors
1 x 9
3 x 3

To factor a perfect square trinomial you need to check that:

1. both a and c are perfect squares
2. the numbers adding to b and multplying to ac are both the same

Short cut:

$$(\sqrt{ax^2} + \sqrt{c})(\sqrt{ax^2} + \sqrt{c})$$

or

$$(\sqrt{ax^2} - \sqrt{c})(\sqrt{ax^2} - \sqrt{c})$$

We take the sign from the middle term!

Examples:

$4x^2 - 12x + 9$

$\sqrt{4x^2} = 2x$

$\sqrt{9} = 3$

Check $2(2x)(3) = 12x$ ✓

$\Rightarrow (2x-3)(2x-3)$

$= (2x-3)^2$

$9x^2 + 6x + 1$

$\sqrt{9x^2} = 3x$

$\sqrt{1} = 1$

Check $2(3x)(1) = 6x$ ✓

$\Rightarrow (3x+1)(3x+1)$

$= (3x+1)^2$