Warm Up
Find the reciprocal of the following functions.

a) $f(x)=4$ $=\frac{1}{4}$

b) $f(x)=-6$ $=-\frac{1}{6}$

c) $f(x)=x$ $=\frac{1}{x}$

d) $f(x)=x+2$ $=\frac{1}{x+2}$

Simplifying Rational Expressions

Lesson objectives

- I know how to graph the reciprocal of a linear function
- I know how to state the restrictions for rational functions

1.1

Lesson objectives | Teachers' notes | Lesson notes

Nelson Page 112 #s 1, 2bc, 5, 6 & 12

Recall

Recall the meaning of $(4)^{-1}$

The reciprocal of 4

Therefore, an exponent of -1 means: Reciprocal

$f^{-1}(x)$ represents: Inverse of $f(x)$

$[f(x)]^{-1}$ represents: Reciprocal of $f(x)$

Graphing the Reciprocal of a Linear Function

To graph the reciprocal of a linear function we have to look at a couple of important pieces of information

1. The points where f(x) = 1 and f(x) = -1. These points are on the function and the reciprocal.

2. Identify if the function is increasing or decreasing.

If the function is increasing, the reciprocal is decreasing and vice versa.

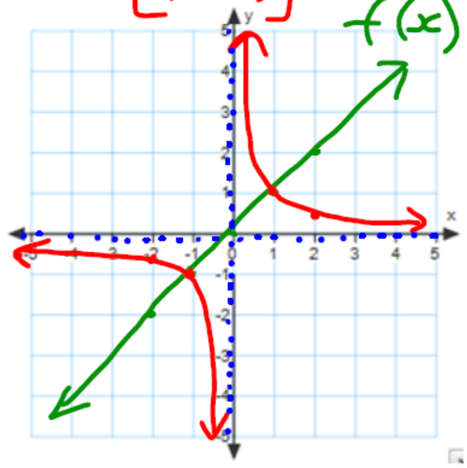
Example:

Example: Graph the function $f(x)$, fill in the table of values, then sketch the function $[f(x)]^{-1}$.

$f(x) = x$

x	$f(x)$	$[f(x)]^{-1}$
-2	-2	-1/2
-1	-1	-1
0	0	X
1	1	1
2	2	1/2

$[f(x)]^{-1}$ $f(x)$



What are the domain and range?

Where do the asymptotes occur? $x = 0$ $y = 0$

Is $f(x)$ increasing or decreasing?

Is $[f(x)]^{-1}$ increasing or decreasing?

$D = \{x \in \mathbb{R}\}$ $D = \{x \in \mathbb{R} \mid x \neq 0\}$

$R = \{y \in \mathbb{R}\}$ $R = \{y \in \mathbb{R} \mid y \neq 0\}$

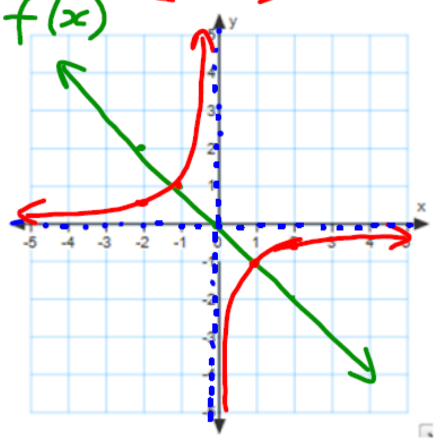
Example:

Example: Graph the function $f(x)$, fill in the table of values, then sketch the function $[f(x)]^{-1}$.

$f(x) = -x$

x	$f(x)$	$[f(x)]^{-1}$
-2	2	1/2
-1	1	1
0	0	X
1	-1	-1
2	-2	-1/2

$f(x)$ $[f(x)]^{-1}$



What are the domain and range?

Where do the asymptotes occur? $x = 0$ $y = 0$

Is $f(x)$ increasing or decreasing?

Is $[f(x)]^{-1}$ increasing or decreasing?

At which point does $f(x) = 1$?

At which point does $f(x) = -1$?

$D = \{x \in \mathbb{R}\}$ $D = \{x \in \mathbb{R} \mid x \neq 0\}$

$R = \{y \in \mathbb{R}\}$ $R = \{y \in \mathbb{R} \mid y \neq 0\}$

Example:

Example: Graph the function $f(x)$, fill in the table of values, then sketch the function $[f(x)]^{-1}$.

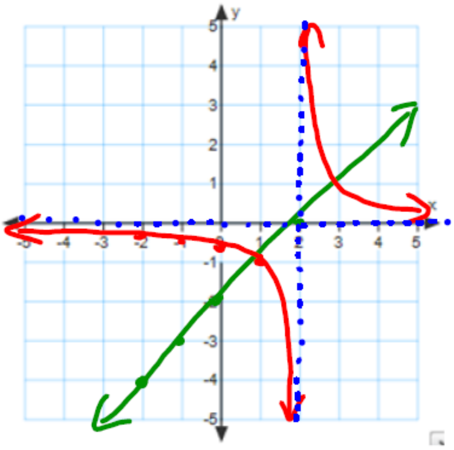
$f(x) = x - 2$

x	$f(x)$	$[f(x)]^{-1}$
-2	-4	$-\frac{1}{4}$
-1	-3	$-\frac{1}{3}$
0	-2	$-\frac{1}{2}$
1	-1	-1
2	0	$\times$

What are the domain and range?

Where do the asymptotes occur?

$x = 2$
$y = 0$



Is $f(x)$ increasing or decreasing?

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Learning Check!

Determine where the vertical asymptotes will occur in the reciprocal function.

a) y = 3x-1 b) y = (3x-1)(x-2) c) y = x² - 3x - 18

a)

$$\frac{1}{3x-1}$$

$$3x - 1 = 0$$

$$\frac{3x}{3} = \frac{1}{3}$$

$$x = \frac{1}{3}$$

b)

$$\frac{1}{(3x-1)(x-2)}$$

$$(3x-1)(x-2) = 0$$

$$x = \frac{1}{3} \quad x = 2$$

c)

$$\frac{1}{(x^2-3x-18)}$$

$$\frac{1}{(x-6)(x+3)}$$

$$(x-6)(x+3) = 0$$

$$x = 6 \quad x = -3$$

What is a Rational Function?

Any function that is the ratio of two polynomials.

A rational function can be expressed as $f(x) = \frac{R(x)}{S(x)}$

where R(x) and S(x) are polynomials and S(x) ≠ 0

For example: $f(x) = \frac{x^2 - 2x + 3}{4x - 1}, x \neq \frac{1}{4}$

What is a restriction?

The values of the variable(s) in a rational function or expression that cause the function to be undefined.

These are the zeros of the denominator or, equivalently, the numbers that are not in the domain of the function.

$\frac{4x+3}{xy}$

$xy \neq 0$

$\Rightarrow x \neq 0$

$y \neq 0$

$\frac{3p - 2q}{p - q}$

$p - q \neq 0$

$\Rightarrow p \neq q$

$\frac{4x+1}{3x-2}$

$3x - 2 \neq 0$

$\frac{3x}{3} \neq \frac{2}{3}$

$x \neq \frac{2}{3}$

Four types of denominator

1. When the denominator is a number (not zero) this leads to no restrictions.

Eg $\frac{3x}{2}$ or $\frac{6y}{5}$

2. The denominator is a monomial (single term) then each variable ≠ 0.

Eg $\frac{6x}{2xy}$ or $\frac{5y}{4x}$

Four types of denominator

3. The denominator is a "relative value". This means that one variable will be expressed in terms of another variable. Set the denominator to zero and solve for one of the variables to find the restriction.

Eg $\frac{3p+q}{p-q}$ or $\frac{5p-6q}{p+q}$

4. The denominator is a binomial or higher order. Factor, set the factors equal to zero and solve to find the restriction(s).

Eg $\frac{7}{4x-1}$ or $\frac{3x-2}{(x+1)(x-2)}$

What are asymptotes?

A vertical asymptote is formed by the restrictions on the domain. We find these by equating the denominator to 0 and solving for x.

A horizontal asymptote occurs where there is a value that we cannot calculate for y (the "c" value). These occur in two cases:

What are asymptotes?

1. The degree of the numerator is equal to the degree of the denominator. In this case the horizontal asymptote is equal to the ratio of the coefficients of the terms with the highest degree.

2. The degree of the numerator is less than the degree of the denominator. In this case the horizontal asymptote is equal to zero.

Example

State the restrictions.

a) $P(n)=\dfrac{3n^3-3n^2}{8n^3-12n^2+4n}$ b) $\dfrac{10x^4-8x^2+4x}{2x^2}$ c) $\dfrac{30x^4y^2}{-6x^7y}$

Restrictions are when denominator = 0

$8n^3-12n^2+4n=0$

$4n(2n^2-3n+1)=0$

$4n(2n-1)(n-1)=0$

$\Rightarrow n \neq 0, \frac{1}{2}, 1$

$\dfrac{2x^2}{2}=\dfrac{0}{2}$

$x^2=0$

$x=\sqrt{0}$

$x \neq 0$

$\dfrac{-6x^7y}{-6}=\dfrac{0}{-6}$

$x^7y=0$

$(x^7)(y)=0$

$x \neq 0$

$y \neq 0$

Simplifying Rational Expressions

Rational expressions can be simplified by factoring the numerator and denominator and then dividing both by their greatest common factor.

Restrictions must always be stated before dividing!

Factor
Restrictions
Simplify

FRS

Example:
Simplify

a) $\dfrac{8x^4y^3z^7}{2xy^2z^5}$

$x \neq 0$
$y \neq 0$
$z \neq 0$

Simplify using exponent laws

$= 4x^3yz^2$

b) $-\dfrac{5x^3y^2}{10xy^3}$

$x \neq 0, y \neq 0$

$= -\dfrac{x^2}{2y}$

c) $\dfrac{5(x+3)}{(x+3)(x-3)}$

$x \neq -3, 3$

$= \dfrac{5\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$

$= \dfrac{5}{(x-3)}$

d) $\dfrac{5x^2+x-4}{25x^2-40x+16}$

$x \neq \frac{4}{5}$

$= \dfrac{\cancel{(5x-4)}(x+1)}{\cancel{(5x-4)}(5x-4)}$

$= \dfrac{x+1}{5x-4}$