

Solutions

1. What is the probability that a hand of five cards contains only

- a) hearts?
- b) red cards?
- c) face cards?

All outcomes for 5 cards from a deck = $52C5$

$$a) n(\text{All Hearts}) = 13C5$$

$$P(\text{All Hearts}) = \frac{13C5}{52C5} = \frac{1287}{2598960} = \frac{33}{66640} \quad (0.0495\%)$$

$$b) n(\text{All Red}) = 26C5$$

$$P(\text{All Red}) = \frac{26C5}{52C5} = \frac{65780}{2598960} = \frac{1265}{49980} \quad (2.531\%)$$

$$c) n(\text{All Face}) = 12C5$$

$$P(\text{All Face}) = \frac{12C5}{52C5} = \frac{792}{2598960} = \frac{33}{108290} \quad (0.0305\%)$$

2. In the story of the Three Little Pigs, the big bad wolf was able to blow down the two houses made of straw and of sticks, but not the house made of bricks. If the wolf chose two houses at random, what is the probability that it would be able to blow them both down?

$$\text{All combinations} = 3C_2$$

$$\text{Combinations to blow both down} = 2C_2$$

$$P(\text{Blow both down}) = \frac{2C_2}{3C_2} = \frac{1}{3}$$

3. A department has 10 employees. Two will be chosen at random to attend a conference. What is the probability that both Sarah and Dan will be selected?

A $\frac{1}{5}$

B $\frac{1}{45}$

C $\frac{2}{45}$

D $\frac{1}{90}$

$$n(\text{Ways to select 2 from 10}) = 10C_2$$

$$n(\text{Select Dan and Sarah}) = 2C_2$$

$$P(\text{Dan and Sarah selected}) = \frac{2C_2}{10C_2} = \frac{1}{45} \Rightarrow B$$

4. To win the grand prize in a hospital lottery, you must match six different numbers chosen from the numbers 1 to 45. What is the probability of winning the grand prize?

A $\frac{1}{5\,864\,443\,200}$

B $\frac{6}{5\,864\,443\,200}$

C $\frac{1}{8\,145\,060}$

D $\frac{6}{8\,145\,060}$

$$n(\text{arrangements}) = 45C_6$$

$$n(\text{winning arrangements}) = 6C_6$$

$$P(\text{winning}) = \frac{6C_6}{45C_6} = \frac{1}{8,145,060}$$

6. In the game of hearts, each of four players receives 13 cards. What is the probability that each player receives 13 cards of the same suit?

$$n(\text{arrangements of 13 card hand}) = 52C_{13}$$

$$n(\text{all 13 from one suit}) = 13C_{13}$$

Need all 4 hands to be of the same suit

$$\Rightarrow P(4 \text{ hands of same suit})$$

$$= \frac{13C_{13} \times 13C_{13} \times 13C_{13} \times 13C_{13}}{52C_{13}}$$

$$= \frac{1 \times 1 \times 1 \times 1}{6.350135596 \times 10^{11}}$$

$$\approx 1.5748 \times 10^{-12}$$

7. Six girls and five boys wish to join a committee. Four of them will be selected. What is the probability that three girls and one boy will be selected?

$$n(\text{arrangements}) = {}_{11}C_4$$

choose 4 from
a total of 11

$$n(3 \text{ girls and } 1 \text{ boy}) = {}_6C_3 \times {}_5C_1$$

$$\begin{aligned} P(3 \text{ girls and } 1 \text{ boy}) &= \frac{{}_6C_3 \times {}_5C_1}{{}_{11}C_4} \\ &= \frac{20 \times 5}{330} \\ &= \frac{100}{330} \\ &= \frac{10}{33} \quad (30.303\%) \end{aligned}$$