Probabilities Using Combinations

Lesson objectives

- I can solve probability problems using counting principles

1.1

Lesson objectives | Teachers' notes | Lesson notes

MHR Page 132 #s 1 - 4, 6 & 7

Warm Up

Experienced card players usually consider probability. For example, a player has the king of spades, king of hearts, queen of diamonds, jack of clubs, and 9 of hearts. She needs to determine whether it is more likely that the next card will be another king, for three-of-a-kind, or a 10 for a five-card run (i.e., 9, 10, J, Q, K). How can she determine the likelihood of drawing each card?

The player already has 2 Kings, so there are only two Kings left.
The player has no Tens, so there are still four Tens left.
$\Rightarrow$ She is more likely to draw a Ten than a King. In fact, twice as likely!

To win a particular lottery, your ticket must match five different numbers from 1 to 30, without regard to order.

1. How many winning combinations are there? Only one.

2. How many ways are there of choosing five different numbers from 1 to 30?

$${}_{30}C_5 = 142,506$$

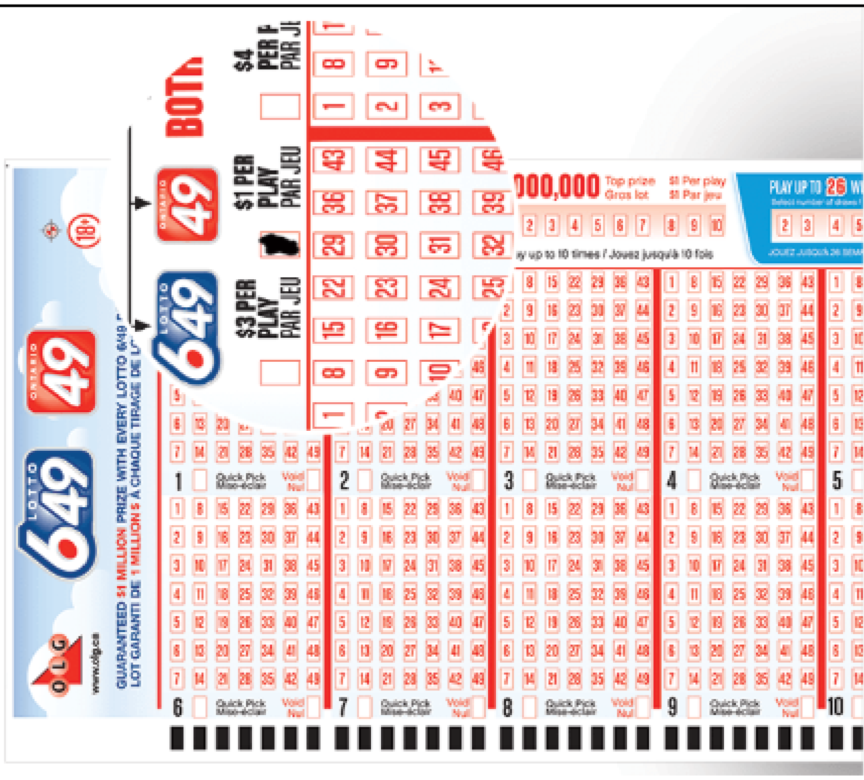
3. What is the probability of winning this lottery?

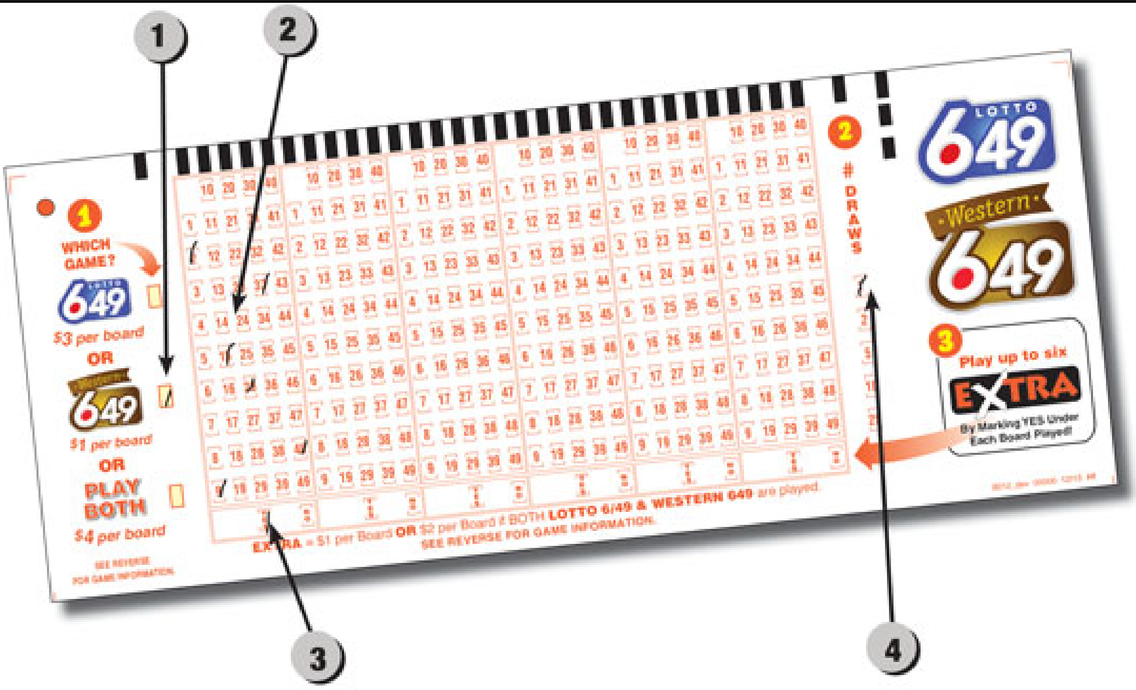
$$\frac{1}{142,506}$$

5. **Extend Your Understanding** Some lotteries sell a specified number of tickets, and the winning ticket is drawn from those sold. Does the probability of winning increase as you buy more tickets?

Yes it does. This is because the number of tickets that are sold is fixed at a value. The more that you buy, the more chances you have to win!

Experiment Time!





Example 1

Chances of Winning

A scratch and win contest at a store allows you to scratch five numbers. If all of your numbers match the winning set of five numbers, chosen from 1 to 25 without regard to order, you win the grand prize.

a) What is the probability of winning the grand prize?

b) To win second prize, four of the five winning numbers must match. What is the probability of winning second prize?

a) Only one way to get the five winning numbers.
There are ${}_{25}C_5$ possible combinations.
$P(\text{win first prize}) = \frac{1}{{}_{25}C_5} = \frac{1}{53,130}$

b) You have ${}_5C_4$ ways to get second prize.
$\Rightarrow P(\text{second prize}) = \frac{{}_5C_4 \times {}_{20}C_1}{{}_{25}C_5}$
$= \frac{5 \times 20}{53,130}$
$= \frac{100}{53,130} = \frac{10}{5313}$

Your Turn

To win the grand prize in a fundraising draw, you need to match seven numbers from 1 to 27, without regard to order.

a) What is the percent chance of winning the grand prize?

b) What is the percent chance of winning second prize, which involves matching six of the seven winning numbers?

c) What is the probability of not winning first or second prize?

d) Comment on how difficult it is to win lottery prizes.

a) $P(\text{winning}) = \frac{{}_7C_7}{{}_{27}C_7} = \frac{1}{888,030}$

$= 0.000112608\%$

b) $P(\text{second}) = \frac{{}_7C_6 \times {}_{20}C_1}{{}_{27}C_7} = \frac{7 \times 20}{888,030}$

$= 0.015765233\%$

c) $P(\text{not } 1^{ST} \text{ or } 2^{ND}) = 100 - P(1^{ST}) - P(2^{ND})$

$= 100 - 0.000112608 - 0.015765233$

$= 99.98310869\ \%$

d) Very, very difficult to win!

Example 2

Choose From Groups

A university task force of 8 people is to be formed from 16 members of the student government and 10 professors. Each person is equally likely to be chosen.

a) What is the probability that there is an equal number of students and professors?

b) What is the probability that at least six members are students?

c) Which outcome is more likely to occur?

Total combinations $= {}_{26}C_8$

a) For equal numbers of each $\Rightarrow$ 4 and 4

$P(\text{equal \#s}) = \frac{{}_{16}C_4 \times {}_{10}C_4}{{}_{26}C_8}$

$= \frac{1820 \times 210}{1,562,275}$

$= \frac{382,200}{1,562,275}$

≈ 0.24464

b) At least 6 are students...

$\Rightarrow$ 6 students 7 students 8 students

$= \frac{{}_{16}C_6 \times {}_{10}C_2}{{}_{26}C_8} + \frac{{}_{16}C_7 \times {}_{10}C_1}{{}_{26}C_8} + \frac{{}_{16}C_8 \times {}_{10}C_0}{{}_{26}C_8}$

$= \frac{360360}{1,562,275} + \frac{114400}{1,562,275} + \frac{12870}{1,562,275}$

$= \frac{487,630}{1,562,275} \approx 0.31213$

c) Of these two options, at least 6 students is more likely than equal numbers of students and professors.

Your Turn

A teacher uses a random name generator to select six students to present their projects. In a class of 23 students, 12 are male and 11 are female.

a) What is the probability that an equal number of male and female students will present?

b) What is the probability that more female than male students will present?

c) Which outcome is more likely?

Total combinations $= {}_{23}C_6$

a) Equal male and female $\Rightarrow$ 3 of each

$$P(\text{equal male and female}) = \frac{{}_{12}C_3 \times {}_{11}C_3}{{}_{23}C_6} = \frac{220 \times 165}{100947} \approx 0.35959$$

b) More female than male when female = 4, 5, or 6

4 Females, 5 Females, 6 Females

$$= \frac{{}_{11}C_4 \times {}_{12}C_2}{{}_{23}C_6} + \frac{{}_{11}C_5 \times {}_{12}C_1}{{}_{23}C_6} + \frac{{}_{11}C_6 \times {}_{12}C_0}{{}_{23}C_6}$$

$$= \frac{330 \times 66}{100947} + \frac{462 \times 12}{100947} + \frac{462 \times 1}{100947}$$

$$= \frac{21780}{100947} + \frac{5544}{100947} + \frac{462}{100947}$$

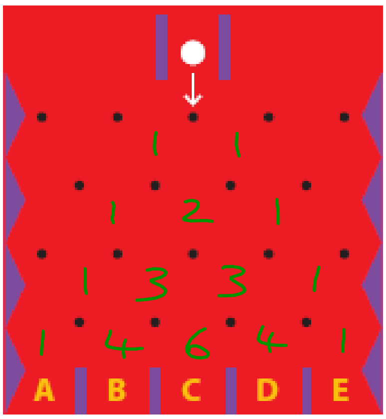
$$= \frac{27786}{100947} \approx 0.27525$$

c) It is more likely that an equal number of males and females present (0.36 vs 0.28)

Example 3

Apply Pascal's Triangle to Probability

In the game of Plinko, a disc is dropped into a slot at the top of a board. When it hits a peg, it falls to the left or right as it travels down the board. State the probability of the disc ending up in each slot at the bottom of the board.



Using Pascal's triangle...

1

1 1

1 2 1

1 3 3 1

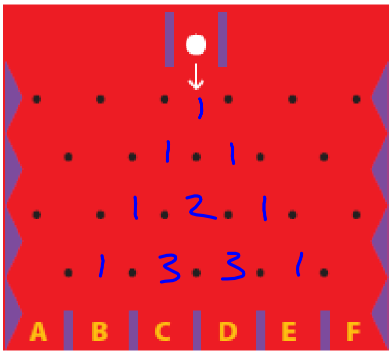
1 4 6 4 1 $\Rightarrow$ 16 routes

$\Rightarrow P(A) = \frac{1}{16}$ $P(B) = \frac{4}{16}$ $P(C) = \frac{6}{16}$

$P(E) = \frac{1}{16}$ $P(D) = \frac{4}{16}$

Your Turn

State the probability of the disc ending up in each slot at the bottom of this Plinko board.



Using Pascal's triangle...

1
1 1
1 2 1
0 1 3 3 1 0
A B C D E F

$\Rightarrow$ 8 routes

$P(A) = 0$
$P(F) = 0$
$P(B) = \frac{1}{8}$
$P(E) = \frac{1}{8}$
$P(C) = \frac{3}{8}$
$P(D) = \frac{3}{8}$

Key Concepts

- You can sometimes use combinations or Pascal's triangle to determine probabilities.
- The numerator, $n(A)$, represents the number of successful outcomes, usually involving restrictions.
- The denominator, $n(S)$, represents the total number of outcomes, with no restrictions.

R2. Tim and Ginny are solving the following problem:

In a race of eight runners, what is the probability that Jake and Hamid are the top two finishers?

Tim solves the problem using permutations: $P(A) = \frac{{}_2P_2}{{}_8P_2}$

Ginny solves it using combinations: $P(A) = \frac{{}_2C_2}{{}_8C_2}$

Are both solutions valid? Why?

Tim is solving for order being important so uses permutations. Ginny is solving for when order is not important. However, in this particular case they both give the same answer (1/28).