Solutions

1. a) Write each term in row 9 of Pascal's triangle using ${}_nC_r$.

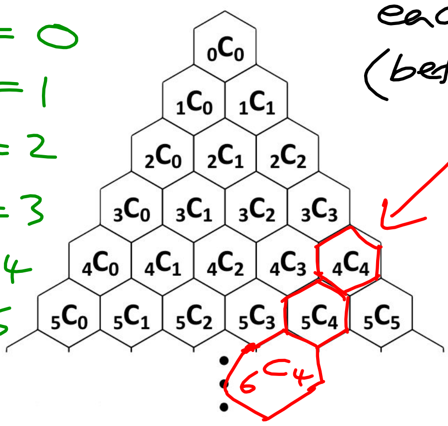
   b) Write the first five terms in diagonal 4 of Pascal's triangle using ${}_nC_r$.

a) ${}_9C_0, {}_9C_1, {}_9C_2, {}_9C_3, {}_9C_4, {}_9C_5, {}_9C_6, {}_9C_7, {}_9C_8, {}_9C_9$

b) ${}_4C_4, {}_5C_4, {}_6C_4, {}_7C_4, {}_8C_4$

diagonal 4 : looking for the 5th term in each row, starting with row 4 (before that, they don't have 5 terms)

$r = 0$
$r = 1$
$r = 2$
$r = 3$
$r = 4$
$r = 5$



Diagonal 4

2. Use Pascal's method to complete the array.

78 286 b

a 1001

c

$a = 78 + 286$
$a = 364$

$286 + b = 1001$
$b = 1001 - 286$
$b = 715$

$a + 1001 = c$
$364 + 1001 = c$
$1365 = c$

3. The first three terms in the expansion of $(x + y)^7$ are

A x^7, x^6y, x^5y^2

B x^7, x^6, x^5

C $xy^7, 7xy^6, 21xy^5$

D $x^7, 7x^6y, 21x^5y^2$

$$(x+y)^7 = {}_7C_0 x^7 y^0,\ {}_7C_1 x^6 y^1,\ {}_7C_2 x^5 y^2$$
$$= (1)(x^7)(1),\ (7)(x^6)(y),\ (21)(x^5)(y^2)$$
$$= x^7,\ 7x^6y,\ 21x^5y^2$$

$\Rightarrow D$

5. Write the first nine rows of Pascal's triangle. Circle the given terms in Pascal's triangle, then circle the correct answer to help use Pascal's method to rewrite each of the following:

a) ${}_5C_2 + {}_5C_3$ **b)** ${}_7C_3 + {}_7C_4$

c) ${}_5C_4 - {}_4C_4$ **d)** ${}_8C_6 - {}_7C_5$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

a) ${}_5C_2 + {}_5C_3$
$= {}_6C_3 = 20$

b) ${}_7C_3 + {}_7C_4$
$= {}_8C_4 = 70$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

c) ${}_5C_4 - {}_4C_4$
$= {}_4C_3 = 4$

d) ${}_8C_6 - {}_7C_5$
$= {}_7C_6 = 7$

6. Communication

a) Evaluate each of the following:

i) ${}_2C_2 + {}_3C_2$

ii) ${}_3C_2 + {}_4C_2$

iii) ${}_4C_2 + {}_5C_2$

b) Describe the results.

c) Identify the terms from part a) in Pascal's triangle.

d) Summarize the results, in general, as they apply to combinations and Pascal's triangle.

a) (i) ${}_2C_2 + {}_3C_2$
$= 1 + 3$
$= 4$

(ii) ${}_3C_2 + {}_4C_2$
$= 3 + 6$
$= 9$

(iii) ${}_4C_2 + {}_5C_2$
$= 6 + 10$
$= 16$

b) They are all perfect squares

c) All of the terms are in diagonal 2

d) These are the sum of adjacent terms in diagonal 2 of Pascal's triangle.
$n^2 = {}_nC_2 + {}_{n+1}C_2$ where $n > 1$

13. Use Pascal's triangle to expand and simplify. **a)** $(x+y)^8$

$$(x+y)^8 = {}_8C_0x^8y^0 + {}_8C_1x^7y^1 + {}_8C_2x^6y^2$$
$$+ {}_8C_3x^5y^3 + {}_8C_4x^4y^4 + {}_8C_5x^3y^5$$
$$+ {}_8C_6x^2y^6 + {}_8C_7x^1y^7 + {}_8C_8x^0y^8$$
$$= x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3$$
$$+ 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8xy^7 + y^8$$

13. Use Pascal's triangle to expand and simplify. **b)** $(x-y)^5$

$$(x-y)^5 = {}_5C_0x^5y^0 + {}_5C_1x^4(-y)^1 + {}_5C_2x^3(-y)^2$$
$$+ {}_5C_3x^2(-y)^3 + {}_5C_4x^1(-y)^4 + {}_5C_5x^0(-y)^5$$
$$= x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3$$
$$+ 5xy^4 - y^5$$

13. Use Pascal's triangle to expand and simplify. **c)** $(2a+b)^4$

$$(2a+b)^4 = {}_4C_0(2a)^4(b)^0 + {}_4C_1(2a)^3(b)^1 + {}_4C_2(2a)^2(b)^2 + {}_4C_3(2a)^1(b)^3 + {}_4C_4(2a)^0(b)^4$$

$$= (1)(16a^4)(b^0) + (4)(8a^3)(b^1) + 6(4a^2)(b^2) + 4(2a)(b^3) + (1)(2^0a^0)(b^4)$$

$$= 16a^4 + 32a^3b + 24a^2b^2 + 8ab^3 + b^4$$