Solutions

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1. Determine the rate of change for each of the following linear relations.

a) A band's number of monthly listeners on a music streaming platform increases by 1000 every month.

b) The height of an elevator decreases by 150 feet every minute.

c) A car travels 240 km every 3 hours.

a) 1000 listeners per month

b) −150 feet per minute

c) $\dfrac{240}{3} = 80$ km/h

1. Determine the rate of change for each of the following linear relations.

d)

Time (h)	Distance (km)
1	80
2	160
3	240
4	320
5	400

e)

Number of Years	Number of People
0	3000
1	4000
2	5000
3	6000
4	7000

f)

Time (min)	Temperature (°C)
0	90
1	75
2	60
3	45
4	30

$$= \frac{80}{1}$$

$$= 80 \text{ km/h}$$

$$= \frac{1000}{1}$$

$$= 1000 \text{ people/year}$$

$$= \frac{-15}{1}$$

$$= -15 \text{ °C/min}$$

1. Determine the rate of change for each of the following linear relations.

g)

$$= \frac{4}{2}$$

$$= 2$$

$$= \$2/\text{ride}$$

h)

$$= \frac{16}{2}$$

$$= 8$$

$$= 8 \text{ m}^2/\text{min}$$

i)

$$= \frac{20}{4}$$

$$= 5$$

$$= 5 \text{ kPa/s}$$

j)

$$= \frac{-1500}{2}$$

$$= -750$$

$$= -\$750/\text{year}$$

2
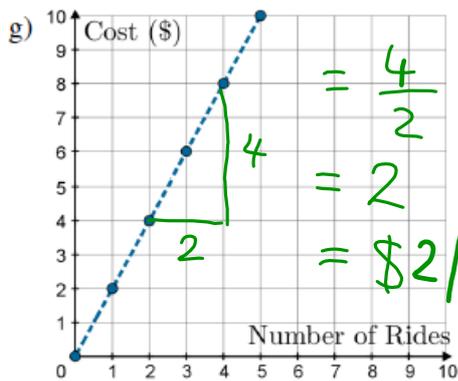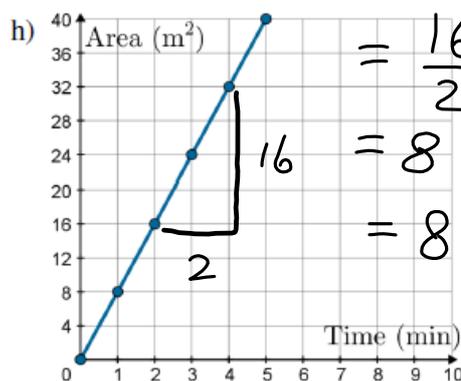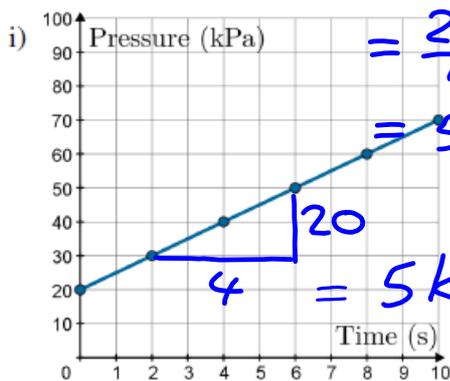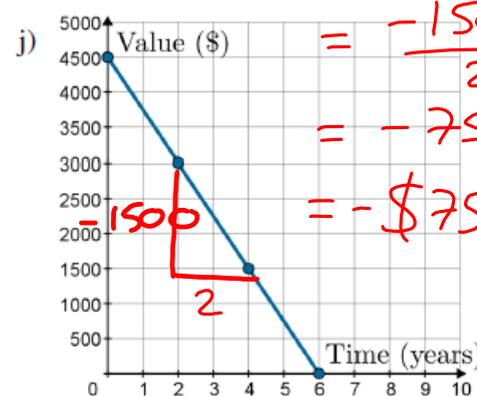
3. The graph on the right shows the volume of water in a tank over time.
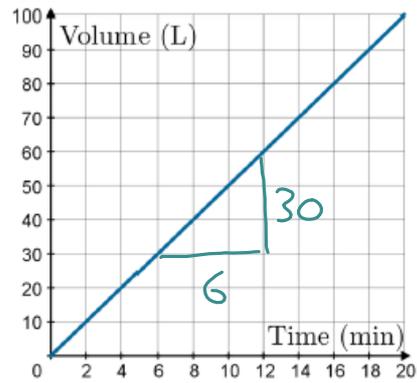
   a) Identify the dependent and independent variables in this relation.

   b) What scale is being used for the time axis?

   c) What scale is being used for the volume axis?

   d) Is the relationship between volume and time linear or non-linear? Explain.

   e) Determine the rate of change of volume in litres per minute.

a) Independent (time), dependent (volume)

b) 2 minutes per square

c) 10 litres per square

d) Linear, because the graph is a straight line

e) $= \dfrac{30}{6} = 5$ litres/min

4. For each of the following linear relations,

   i) state the initial value (start value).   ii) find the rate of change.

a)

Age (years)	Mass (g)
0	0
2	50
4	100
6	150
8	200

b)

Time (min)	Distance (m)
0	250
20	210
40	170
60	130
80	90

c)

Time (s)	Area (cm²)
30	80
60	152
90	224
120	296
150	368

Initial value when independent variable $= 0$

Rate of change $= \dfrac{\text{change in } y}{\text{change in } x}$

a) initial value $= 0$ g

   R of C $= \dfrac{50}{2}$

   $= 25$ g/year

b) initial value $= 250$ m

   R of C $= \dfrac{-40}{20}$

   $= -2$ m/min

c) initial value $= 8$ cm²

   R of C $= \dfrac{72}{30}$

   $= 2.4$ sec/cm²

3

**4.** For each of the following linear relations,

  i) state the initial value (start value).   ii) find the rate of change.

d)

Savings ($) vs Days Worked graph, with marks 300 and 15.

Initial value = $200

$$R\ of\ C = \frac{300}{15}$$

$$= \$20/day$$

e)

Speed (knots) vs Time (min) graph, with marks -150 and 12.
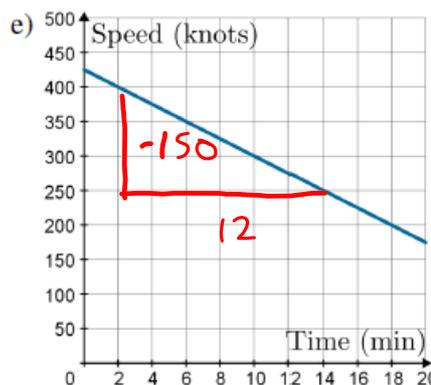
Initial value = 425 knots

$$R\ of\ C = \frac{-150}{12}$$

$$= -12.5\ knots/min$$

7. For each relation, if the rate of change is constant, determine its value (without units).
  Otherwise, state whether the rate of change is increasing or decreasing.

a)

Graph of y vs x, with marks 12 and 12.
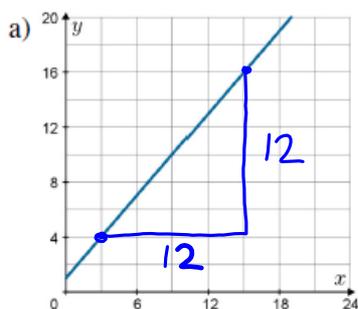
Straight line
⟹ Constant rate of change

$$= \frac{12}{12}$$

$$= 1$$

b)

Graph of y vs x, curved.
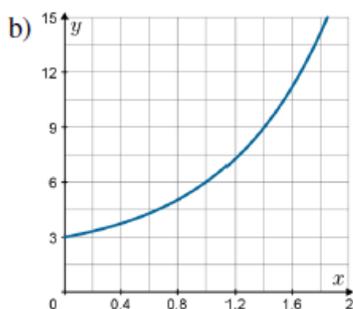
Rate of change is increasing because the line is getting steeper.

c)

Graph of A vs t, with marks -3 and 2.
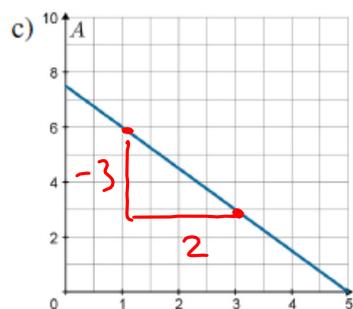
Straight line
⟹ constant rate of change

$$= \frac{-3}{2}$$

$$= -1.5$$

7. For each relation, if the rate of change is constant, determine its value (without units).
Otherwise, state whether the rate of change is increasing or decreasing.

d)

e)

f)
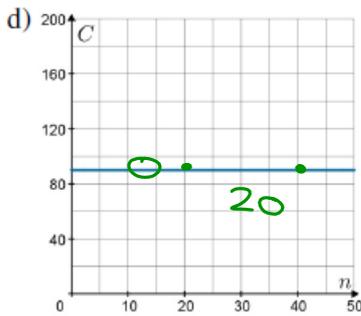
Straight line
$\Rightarrow$ constant
rate of change

$= \dfrac{0}{20}$

$= 0$
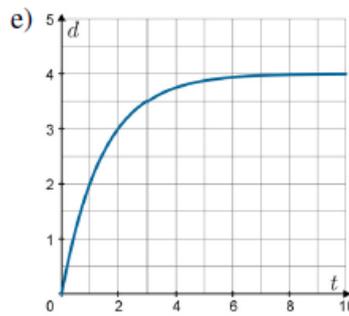
Rate of change
is decreasing
because the
line is becoming
less steep.
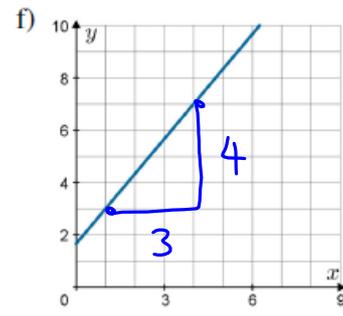
Straight line
$\Rightarrow$ constant
rate of change

$= \dfrac{4}{3}$

$= 1\frac{1}{3}$

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7. For each relation, if the rate of change is constant, determine its value (without units).
Otherwise, state whether the rate of change is increasing or decreasing.

g)

x	y
0	−180
15	−138
30	−96
45	−54
60	−12

15 $\rightarrow$ −42
15 $\rightarrow$ −42
15 $\rightarrow$ −42
15 $\rightarrow$ −42

1ST differences
are constant
$\Rightarrow$ constant
rate of change

$= \dfrac{-42}{15}$

$= \dfrac{-14}{5}$

$= -2.8$

h)

t	h
0	1510.0
2	1686.4
4	1823.6
6	1921.6
8	1980.4

$\rightarrow$ 176.4
$\rightarrow$ 137.2
$\rightarrow$ 98
$\rightarrow$ 58.8

Rate of change
is decreasing
because the
first differences
are getting
smaller.

i)

n	C
0	5.2
1.25	0.7
2.5	−3.8
3.75	−8.3
5	−12.8

$\rightarrow$ −4.5
$\rightarrow$ −4.5
$\rightarrow$ −4.5
$\rightarrow$ −4.5

1ST differences
are constant
$\Rightarrow$ constant
rate of change

$= \dfrac{-4.5}{1.25}$

$= -3.6$

8. The graph on the right shows the volume of liquid in two tanks as they are drained.

a) How much liquid is initially in each tank?

b) How long does it take each tank to drain?

c) Determine the rate of change of volume for the liquid in each tank.

d) Explain why the tank with the lower rate of change value is actually draining more quickly than the other tank.
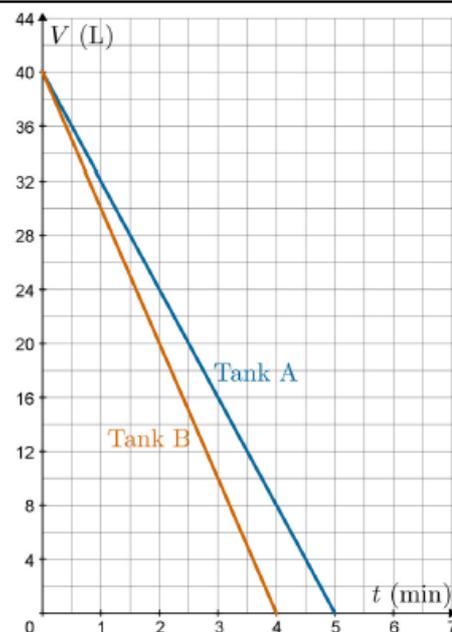
a) 40 litres

b) Tank A takes 5 mins
Tank B takes 4 mins

c) $Tank_A = \frac{-40}{5} = -8$ L/min

$Tank_B = \frac{-40}{4} = -10$ L/min

d) Negative rate indicates the direction of flow, not the speed of it.

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e) When initially full, a third tank, Tank C, takes twice as long as Tank B to fully drain and drains at half the rate of Tank A. What is the capacity of Tank C?

Tank B takes 4 minutes to drain
$\Rightarrow$ Tank C must take 8 minutes

Tank A rate is -8 L/min
$\Rightarrow$ Tank C rate is -4 L/min

Capacity of $Tank_C$ = Rate × Time
= -4×8
= -32 $\Rightarrow$ 32 litres

Recall: negative is to do with direction.