Solutions

1. How many different sums of money can be made from a $5 bill, a $10 bill, a $20 bill, and a $50 bill?

Must use at least one bill

$$\Rightarrow {}_4C_1 + {}_4C_2 + {}_4C_3 + {}_4C_4$$

$$= 4 + 6 + 4 + 1$$

$$= 15 \text{ different sums}$$

OR   You have 4 bills. You can use all or some of them.

$$\Rightarrow 2^4 - 1$$

use/don't use (2 options)

4 bills

subtract the null set. We can't choose zero bills.

$$= 16 - 1$$
$$= 15 \text{ different sums}$$

2. In how many ways could a group of 10 people form a committee with at least 8 people on it?

   A 45   **B** 56   **C** 450   **D** 1016

At least 8 from 10

$\Rightarrow$ 8 people OR 9 people OR 10 people

$$_{10}C_8 + {}_{10}C_9 + {}_{10}C_{10}$$

$$= 45 + 10 + 1$$

$$= 56 \text{ ways} \Rightarrow B$$

3. If a set has 12 elements, how many subsets can be formed?

   A 12   **B** 24   **C** 4095   **D** 4096

Must choose something so the null set is not included.

$$\Rightarrow {}_{12}C_1 + {}_{12}C_2 + {}_{12}C_3 + \dots + {}_{12}C_{11} + {}_{12}C_{12}$$

$$= 4095 \Rightarrow C$$

OR   use the indirect method

12 elements can be either chosen or not chosen

$$\Rightarrow 2^{12} - 1$$

choose/not choose   number of elements   subtract the null set

$$= 4096 - 1$$
$$= 4095 \Rightarrow C$$

4. A judging panel will have 6 members chosen from 8 teachers and 10 students. There must be at least 3 students on the panel. In how many ways could there be

a) 3 students on the panel?
b) 4 students on the panel?
c) 5 students on the panel?
d) least 3 students on the panel?

a) 3 students $\Rightarrow$ 3 teachers
$$_{10}C_3 \times {_8}C_3 = 120 \times 56 = 6720 \text{ ways}$$

b) 4 students $\Rightarrow$ 2 teachers
$$_{10}C_4 \times {_8}C_2 = 210 \times 28 = 5880 \text{ ways}$$

c) 5 students $\Rightarrow$ 1 teacher
$$_{10}C_5 \times {_8}C_1 = 252 \times 8 = 2016 \text{ ways}$$

d) Only stipulation is at least 3 students
$\Rightarrow$ can have either of (a), (b), (c) or also
$$_{10}C_6 \times {_8}C_0 = 210 \times 1 = 210$$

6 students   0 teachers

Total ways of at least 3 students $=$

$$\begin{array}{r} 6720 \\ 5880 \\ 2016 \\ + \quad 210 \\ \hline 14,826 \end{array}$$

5. **Communication** Identify whether the following situations involve permutations, combinations, or both. Justify your choice.

a) forming a committee of 5 people from a group of 12 people

Combinations — order is not important

b) choosing a president, a vice president, and a treasurer from a committee of 12 members

Permutations — specific roles are assigned, so the order is important

c) choosing 4 men and 4 women to be on a basketball team from among 6 men and 6 women, and assembling the athletes for a team photo

Both — combinations for picking the team, permutations for the positions for the photo

d) naming 3 people from among 15 contestants to win 3 different prizes

Permutations — winning different prizes, so the order is important