

Solutions

4. Expand and simplify.

d) $4(n - 4)(3 + n) - 3(n - 5)(n + 8)$

$$= (4n - 16)(3 + n) - (3n - 15)(n + 8)$$

$$= 12n + 4n^2 - 48 - 16n - [3n^2 + 24n - 15n - 120]$$

$$= 4n^2 - 4n - 48 - [3n^2 + 9n - 120]$$

$$= 4n^2 - 4n - 48 - 3n^2 - 9n + 120$$

$$= n^2 - 13n + 72$$

4. Expand and simplify.

e) $3(2x - 1)^2 - 5(4x + 1)^2$

$$\begin{aligned}
 &= 3(2x-1)(2x-1) - 5(4x+1)(4x+1) \\
 &= 3(4x^2 - 2x - 2x + 1) - 5(16x^2 + 4x + 4x + 1) \\
 &= 3(4x^2 - 4x + 1) - 5(16x^2 + 8x + 1) \\
 &= 12x^2 - 12x + 3 - 80x^2 - 40x - 5 \\
 &= -68x^2 - 52x - 2
 \end{aligned}$$

4. Expand and simplify.

f) $2(3a + 4)(a - 6) - (3 - a)^2 + 4(5 - a)$

$$\begin{aligned}
 &= (6a+8)(a-6) - (3-a)(3-a) + 20 - 4a \\
 &= 6a^2 - 36a + 8a - 48 - [9 - 3a - 3a + a^2] + 20 - 4a \\
 &= 6a^2 - 28a - 48 - 9 + 6a - a^2 + 20 - 4a \\
 &= 5a^2 - 26a - 37
 \end{aligned}$$

6. Determine whether each pair of expressions is equivalent.

a) $(3x - 2)(2x - 1)$ and $3x(2x - 1) - 2(2x - 1)$

$$\begin{aligned}
 &= 6x^2 - 3x - 4x + 2 & &= 6x^2 - 3x - (4x - 2) \\
 &= 6x^2 - 7x + 2 & &= 6x^2 - 3x - 4x + 2 \\
 & & &= 6x^2 - 7x + 2
 \end{aligned}$$

\Rightarrow Yes, they are equivalent

6. Determine whether each pair of expressions is equivalent.

c) $(x + 2)(3x - 1) - (1 - 2x)^2$ and $x^2 + 9x - 3$

$$\begin{aligned}
 &= 3x^2 - x + 6x - 2 - (1 - 2x)(1 - 2x) \\
 &= 3x^2 + 5x - 2 - (1 - 2x - 2x + 4x^2) \\
 &= 3x^2 + 5x - 2 - 1 + 4x - 4x^2 \\
 &= -x^2 + 9x - 3
 \end{aligned}$$

\Rightarrow No, they are not equivalent

6. Determine whether each pair of expressions is equivalent.

e) $(4x + y - 3)^2$ and $16x^2 - 8xy + 24x + y^2 - 6y + 9$

	$4x$	y	-3
$4x$	$16x^2$	$4xy$	$-12x$
y	$4xy$	y^2	$-3y$
-3	$-12x$	$-3y$	9

$$= 16x^2 + 8xy - 24x + y^2 - 6y + 9$$

\Rightarrow No, they are not equivalent

7. Is the equation $(x - 1)(x^4 + x^3 + x^2 + x + 1) = x^5 - 1$ true for all, some, or no real numbers? Explain.

Show they are equivalent (or not)

	x^4	x^3	x^2	x	1
x	x^5	x^4	x^3	x^2	x
-1	$-x^4$	$-x^3$	$-x^2$	$-x$	-1

$$= x^5 - 1$$

\Rightarrow They are equivalent, so it is true for all numbers.

11. Expand and simplify.

a) $(x^2 + 2x - 1)^2$ c) $(x^3 + x^2 + x + 1)(x^3 - x^2 - x - 1)$

	x^2	$2x$	-1	
x^2	x^4	$2x^3$	$-x^2$	
$2x$	$2x^3$	$4x^2$	$-2x$	
-1	$-x^2$	$-2x$	1	

$$= x^4 + 4x^3 + 2x^2 - 4x + 1$$

	x^3	x^2	x	1	
x^3	x^6	x^5	x^4	x^3	
$-x^2$	$-x^5$	$-x^4$	$-x^3$	$-x^2$	
$-x$	$-x^4$	$-x^3$	$-x^2$	$-x$	
-1	$-x^3$	$-x^2$	$-x$	1	

$$= x^6 - x^4 - 2x^3 - 3x^2 - 2x + 1$$

11. Expand and simplify.

b) $(2 - a)^3$ d) $2(x + 1)^2 - 3(2x - 1)(3x - 5)$

$$= (2 - a)(2 - a)(2 - a)$$

$$= (4 - 2a - 2a + a^2)(2 - a)$$

$$= (4 - 4a + a^2)(2 - a)$$

$$= 8 - 8a + 2a^2 - 4a + 4a^2 - a^3$$

$$= -a^3 + 6a^2 - 12a + 8$$

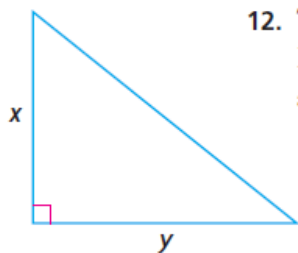
$$= 2(x + 1)(x + 1) - 3(2x - 1)(3x - 5)$$

$$= 2(x^2 + x + x + 1) - 3(6x^2 - 10x - 3x + 5)$$

$$= 2(x^2 + 2x + 1) - 3(6x^2 - 13x + 5)$$

$$= 2x^2 + 4x + 2 - 18x^2 + 39x - 15$$

$$= -16x^2 + 43x - 13$$



12. The two sides of the right triangle shown at the left have lengths x and y . Represent the change in the triangle's area if the length of one side is doubled and the length of the other side is halved.

$$\text{Area} = \frac{1}{2}xy$$

$$\text{Double } x \rightarrow 2x$$

$$\text{Halve } y \rightarrow \frac{y}{2}$$

$$\begin{aligned} \text{Area} &= \frac{1}{2}(2x)\left(\frac{y}{2}\right) \\ &= \frac{1}{2}xy \end{aligned}$$

\Rightarrow Area is unchanged

13. The kinetic energy of an object is given by $E = \frac{1}{2}mv^2$, where m represents the mass of the object and v represents its speed. Write a simplified expression for the kinetic energy of the object if

a) its mass is increased by x

b) its speed is increased by y

$$E = \frac{1}{2}(m+x)v^2$$

$$E = \frac{1}{2}m(v+y)^2$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}xv^2$$

$$E = \frac{1}{2}m(v+y)(v+y)$$

$$E = \frac{1}{2}m(v^2 + vy + vy + y^2)$$

$$E = \frac{1}{2}m(v^2 + 2vy + y^2)$$

$$E = \frac{1}{2}mv^2 + mvy + \frac{1}{2}my^2$$