

# Common Factors

## Lesson objectives

- I know how to common factor a polynomial
- I know how to factor by grouping
- I understand the geometric relationship between factoring and expanding

1.1

Lesson objectives

Teachers' notes

Lesson notes

MHR Page 234 #s 1, 3aceg, 4bdfh, 5, 6, 9,  
13 & 14

Expand the following:

$$\begin{aligned} & 3(x^2 + 2x + 3) \\ &= 3x^2 + 6x + 9 \end{aligned}$$

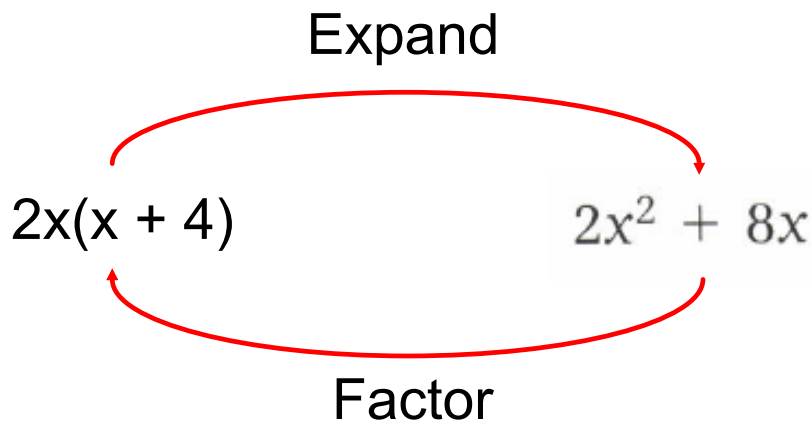
$$\begin{aligned} & 2x(x + 4) \\ &= 2x^2 + 8x \end{aligned}$$

What if we went the other way?

What did I multiply to get:

$$\begin{aligned} 3x^2 + 6x & \Rightarrow 3(x^2 + 2x) & 2x^2 + 4x + 8x^3 & \Rightarrow 2(x^2 + 2x + 4x^3) \\ & \text{These are partially factored} \\ 3x(x + 2) & \Rightarrow 3x(x + 2) & 2x(x + 2 + 4x^2) & \Rightarrow 2x(x + 2 + 4x^2) \\ & \text{These are now fully factored} \end{aligned}$$

## How are expanding and factoring related?



These are the **inverse** (reverse) of each other.

Just like **adding** and subtracting, multiplying and **dividing**, squaring and **square rooting**, expanding and factoring are the **opposite** of each other.

## Common Factoring

When I expand I **multiply**, therefore common factoring is like **dividing**.

We need to find the **greatest common factor** of all terms in the polynomial.

We then **divide** by the GCF from each term and that becomes our **first** factor. What is left after the division is our **second** factor.

To find the GCF we need to find the **largest number** that will divide into each of the terms and we also need the **smallest exponent** that is in each common variable.

**Factor**

$$2x^2 + 12x^1 - 4x^0$$

$$\text{GCF} = 2$$

$$= 2(x^2 + 6x - 2)$$

$$3x^3 + 21x^1$$

$$\text{GCF} = 3x$$

$$= 3x(x^2 + 7)$$

$$6m^1n^1 + 2m^1 + 4m^2$$

$$\text{GCF} = 2m$$

$$= 2m(3n + 1 + 2m)$$

$$3bc^2 + 6c^1 + 9b^3c^2$$

$$\text{GCF} = 3c$$

$$= 3c(bc + 2 + 3b^3c)$$

**A Binomial as a Common Factor**

Divide each term by the common **binomial** and put that in the **second** bracket.

$$\underline{3(x+1)} + \underline{2(x+1)}$$

$$= (x+1)(3+2)$$

$$\underline{x(2x+3)} - \underline{4(2x+3)}$$

$$= (2x+3)(x-4)$$

## Factoring by Grouping

Factor groups of **two terms** with a common factor to produce a **binomial** common factor.

$$\underline{ax} + \underline{ay} + \underline{2x} + \underline{2y}$$

$$= a(x+y) + 2(x+y)$$

$$= (x+y)(a+2)$$

$$\underline{9x^2} + \underline{15x} + \underline{3x} + \underline{5}$$

$$= 3x(3x+5) + 1(3x+5)$$

$$= (3x+5)(3x+1)$$

$$\underline{ax} + \underline{ay} + \underline{2x} + \underline{2y}$$

$$= x(a+2) + y(a+2)$$

$$= (a+2)(x+y)$$

$$\underline{9x^2} + \underline{15x} + \underline{3x} + \underline{5}$$

$$= 3x(3x+1) + 5(3x+1)$$

$$= (3x+1)(3x+5)$$