Common Factors

Lesson objectives

- I know how to common factor a polynomial
- I know how to factor by grouping
- I understand the geometric relationship between factoring and expanding

1.1

Lesson objectives | Teachers' notes | Lesson notes

MHR Page 234 #s 1, 3aceg, 4bdfh, 5, 6, 9, 13 & 14

Expand the following:

$3(x^2+2x+3)$

$= 3x^2 + 6x + 9$

$2x(x+4)$

$= 2x^2 + 8x$

What if we went the other way?

What did I multiply to get:

$3x^2 + 6x$

$\Rightarrow 3(x^2+2x)$

$2x^2 + 4x + 8x^3$

$\Rightarrow 2(x^2+2x+4x^3)$

These are partially factored

$\Rightarrow 3x(x+2)$

$\Rightarrow 2x(x+2+4x^2)$

These are now fully factored

How are expanding and factoring related?

Expand

$2x(x + 4)$ $\qquad\qquad$ $2x^2 + 8x$

Factor

These are the inverse (reverse) of each other.

Just like adding and subtracting, multiplying and dividing, squaring and square rooting, expanding and factoring are the opposite of each other.

Common Factoring

When I expand I multiply, therefore common factoring is like dividing.

We need to find the greatest common factor of all terms in the polynomial.

We then divide by the GCF from each term and that becomes our first factor. What is left after the division is our second factor.

To find the GCF we need to find the largest number that will divide into each of the terms and we also need the smallest exponent that is in each common variable.

Factor

$2x^2 + 12x^1 - 4x^0$

GCF = 2

$= 2(x^2 + 6x - 2)$

$3x^3 + 21x^1$

GCF = 3x

$= 3x(x^2 + 7)$

$6mn^1 + 2m^1 + 4m^2$

GCF = 2m

$= 2m(3n + 1 + 2m)$

$3b^1c^2 + 6c^1 + 9b^3c^2$

GCF = 3c

$= 3c(bc + 2 + 3b^3c)$

A Binomial as a Common Factor

Divide each term by the common binomial and put that in the second bracket.

$3(x+1) + 2(x+1)$

$= (x+1)(3+2)$

$x(2x+3) - 4(2x+3)$

$= (2x+3)(x-4)$

Factoring by Grouping

Factor groups of two terms with a common factor to produce a binomial common factor.

$ax + ay + 2x + 2y$

$= a(x+y) + 2(x+y)$

$= (x+y)(a+2)$

$9x^2 + 15x + 3x + 5$

$= 3x(3x+5) + 1(3x+5)$

$= (3x+5)(3x+1)$

$ax + ay + 2x + 2y$

$= x(a+2) + y(a+2)$

$= (a+2)(x+y)$

$9x^2 + 15x + 3x + 5$

$= 3x(3x+1) + 5(3x+1)$

$= (3x+1)(3x+5)$