MTH1W Grade 9 Mathematics

3.2 What is a Linear Relation?

Goal(s) *- To identify the relationship between two variables as linear*

- To identify a linear relationship from its graph

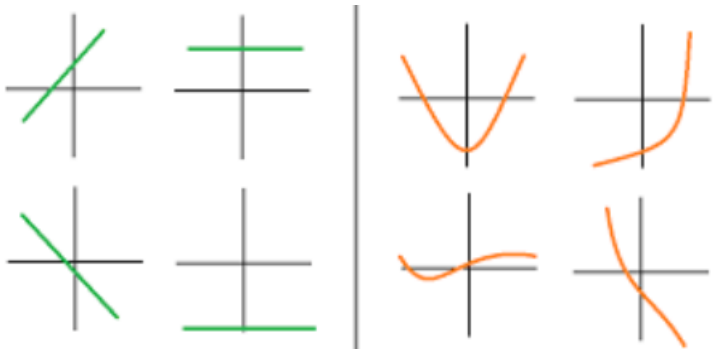
- To use first differences to identify a linear relationship

- To identify the initial value (y-intercept) from a graph

Page 108 #s 1, 3ace, 4, 6, 8, 9, 12

Linear Relation vs. Non-Linear Relation

A linear relation is linear relation has a **constant rate of change**.
When graphed, a linear relationship is a of a **straight line**.



Linear **Non-linear**

The **initial value** of a linear relation is the point where the graph crosses the vertical axis (y-axis). At this point value of the independent variable (usually x) is 0.

A **table of values** is used to record the coordinates of points in a relation.

The **x-coordinate** (independent variable) is listed on the left of the table beginning with the smallest value.

The **y-coordinate** (dependent variable) is listed to the right of the x-coordinate.

If the table is in rows, then the first row would be the x-coordinates and the second row would be the y-coordinates

x	y
-2	6
-1	9
0	12
1	15

Can we determine if a relationship is linear from its table of values?

x	*y*
-3	*9*
-2	*4*
-1	*1*
0	*0*
1	*1*
2	*4*
3	*9*

x	*y*
-3	*-9*
-2	*-5*
-1	*-1*
0	*3*
1	*7*
2	*11*
3	*15*

Yes, we can. We need to look at something called the first differences.

First differences are the differences between consecutive y-values in tables of values with evenly spaced x-values.

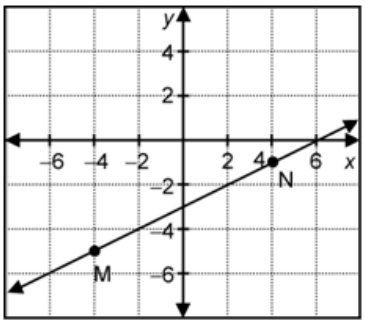
To determine the first differences of an equation, subtract consecutive y-values.

x	y	1st Differences
0	4	
1	7	7 - 4 = 3
2	10	10 - 7 = 3
3	13	13 – 10 = 3
4	16	16 – 13 = 3

1ST differences are the same
⟹ linear relation

If the first differences of a relation are **constant**, the relation is **linear**.

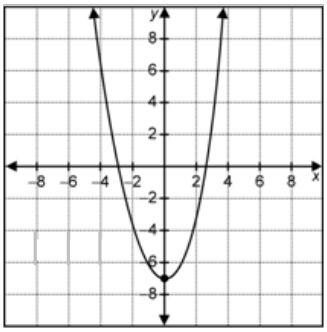
x	y	1st Diff.
-2	-4	
-1	-3.5	-3.5 - (-4) = **0.5**
0	-3	-3 - (-3.5) = **0.5**
1	-2.5	-2.5 - (-3) = **0.5**



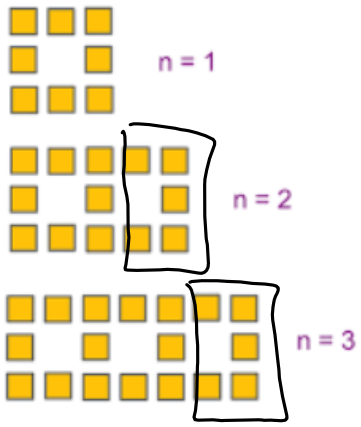
Straight line
graph → linear

If the first differences of a relation are **not constant**, the relation is **non-linear**.

x	y	1st Diff.
-1	-6	
0	-7	-7 - (-6) = **-1**
1	-6	-6 - (-7) = **1**
2	-3	-3 - (-6) = **3**



Curved line
graph → non-linear



Complete the table of values.
Is this relationship linear?
What would the initial value be?
How many squares would be in the 12th figure?

Figure #	# of Squares
1	8
2	13
3	18
4	23
5	28
6	33

12^{th} figure

$= 12 \times 5 + 3$

1st diff

initial value

Yes, the relationship is linear

Initial value would be 3. This is because we are looking for the number of squares when n=0. Counting back 5 from 8 gives 3.

The **cost** of a taxi ride is ***$5.00*** plus ***$0.75*** for every kilometre travelled.

Is the relationship between the distance travelled and the total cost linear or non-linear?

Complete the table of values.

Determine the total cost of ***25km*** ride.

Determine the distance travelled if the total cost is ***$19.25***.

Distance	Cost
0	5
3	7.25
6	9.5
9	11.75
12	14

Linear, because cost per km is the same

25km $\Rightarrow 25 \times 0.75 + 5$
$= 18.75 + 5$
$= \$23.75$

$19.25 $\Rightarrow (19.25 - 5) \div 0.75$
$= 14.25 \div 0.75$
$= 19km$

Do the reverse process