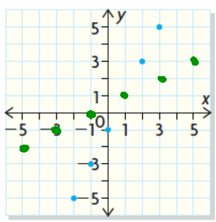
Solutions

2. Copy the graph of each function and graph its inverse. For each graph, identify the points that are common to the function and its inverse. Which inverse relations are functions?

a)

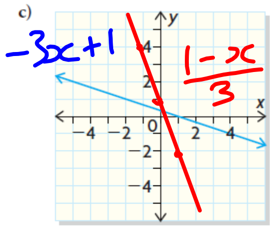


$(-2,-5) \rightarrow (-5,-2)$

$(-1,-3) \rightarrow (-3,-1)$ etc.

$(1,1)$ is common to both. Inverse is a function.

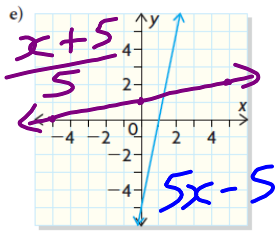
c)



$\left(\frac{1}{4}, \frac{1}{4}\right)$ is common to both.

Inverse is a function.

e)



$\frac{x+5}{5} = 5x-5$

$x+5 = 25x-25$

$-24x = -30$

$\frac{-24x}{-24} = \frac{-30}{-24}$

$x = 1\frac{1}{4}$

Sub into to either to find y

$\Rightarrow \left(1\frac{1}{4}, 1\frac{1}{4}\right)$

Inverse is a function.

3. Determine whether each pair of functions described in words are inverses.

a) f: Multiply by 3, then add 1; g: Divide by 3, then subtract 1.

b) f: Multiply by 5, then subtract 2; g: Add 2, then divide by 5.

a) $f(x) = 3x+1$ $g(x) = \frac{x}{3}+1$

$y = 3x+1$

$x = 3y+1$

$\frac{x-1}{3} = \frac{3y}{3}$

$\neq g(x)$ $\Rightarrow$ Not inverses

b) $f(x) = 5x-2$ $g(x) = \frac{x+2}{5}$

$y = 5x-2$

$x = 5y-2$

$\frac{x+2}{5} = \frac{5y}{5}$

$= g(x)$ $\Rightarrow$ Are inverses

6. Determine the inverse of each linear function by interchanging the variables.

b) $f(x) = 2 - x$ **d)** $f(x) = -\frac{1}{5}x - 2$ **f)** $f(x) = \frac{x-3}{4}$

b)

$y = 2-x$

$x = 2-y$

$\frac{x-2}{-1} = \frac{-y}{-1}$

$2-x = y$

$f^{-1}(x) = 2-x$

d)

$y = -\frac{1}{5}x-2$

$x = -\frac{1}{5}y-2$

$x+2 = -\frac{1}{5}y$

$-5(x+2) = y$

$f^{-1}(x) = -5(x+2)$

f)

$y = \frac{x-3}{4}$

$x = \frac{y-3}{4}$

$4x = y-3$

$4x+3 = y$

$f^{-1}(x) = 4x+3$

9. a) Determine f^{-1} for the linear function $f(x) = 5x - 2$.
b) Graph f and f^{-1} on the same axes.
c) Explain how you can tell that f^{-1} is also a linear function.
d) State the coordinates of any points that are common to both f and f^{-1}.
e) Compare the slopes of the two lines.
f) Repeat parts (a) to (e) for $g(x) = -\frac{1}{2}x + 3$, $h(x) = 2x - 1$, $p(x) = 6 - x$, and $q(x) = 2$.

$y = 5x - 2$

$x = 5y - 2$

$\frac{x+2}{5} = \frac{5y}{5}$

$f^{-1}(x) = \frac{x+2}{5}$

Straight line graph

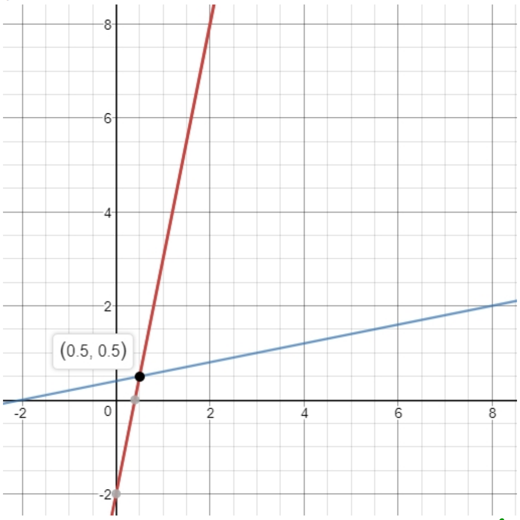
$5x - 2 = \frac{x+2}{5}$

$25x - 10 = x + 2$

$\frac{24x}{24} = \frac{12}{24}$

$x = \frac{1}{2}$

$\Rightarrow \left(\frac{1}{2}, 2\frac{1}{2}\right)$



Slopes are the reciprocal of each other (5 vs $\frac{1}{5}$)

9. a) Determine f^{-1} for the linear function $f(x) = 5x - 2$.
b) Graph f and f^{-1} on the same axes.
c) Explain how you can tell that f^{-1} is also a linear function.
d) State the coordinates of any points that are common to both f and f^{-1}.
e) Compare the slopes of the two lines.
f) Repeat parts (a) to (e) for $g(x) = -\frac{1}{2}x + 3$, $h(x) = 2x - 1$, $p(x) = 6 - x$, and $q(x) = 2$.

$y = -\frac{1}{2}x + 3$

$x = -\frac{1}{2}y + 3$

$x - 3 = -\frac{1}{2}y$

$\frac{2x - 6}{-1} = \frac{-y}{-1}$

$-2x + 6 = y$

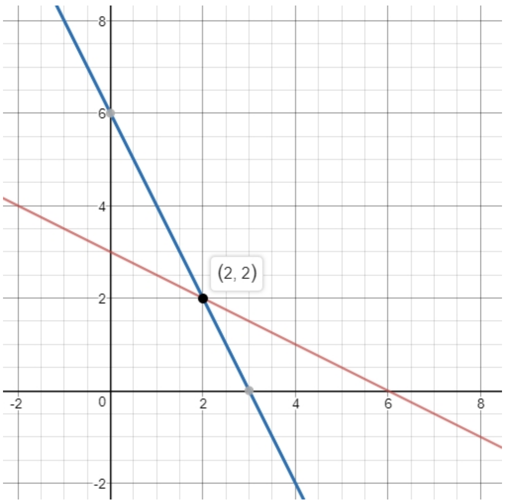
$g^{-1}(x) = -2x + 6$

Straight line graph

$-\frac{1}{2}x + 3 = -2x + 6$

$\frac{1\frac{1}{2}x}{1\frac{1}{2}} = \frac{3}{1\frac{1}{2}}$

$x = 2 \Rightarrow (2, 2)$



Slopes are the reciprocal of each other ($-\frac{1}{2}$ vs -2)

9. a) Determine f^{-1} for the linear function $f(x) = 5x - 2$.
b) Graph f and f^{-1} on the same axes.
c) Explain how you can tell that f^{-1} is also a linear function.
d) State the coordinates of any points that are common to both f and f^{-1}.
e) Compare the slopes of the two lines.
f) Repeat parts (a) to (e) for $g(x) = -\frac{1}{2}x + 3$, $h(x) = 2x - 1$, $p(x) = 6 - x$, and $q(x) = 2$.

$y = 2x - 1$

$x = 2y - 1$

$\frac{x+1}{2} = \frac{2y}{2}$

$h^{-1}(x) = \frac{x+1}{2}$

Straight line graph

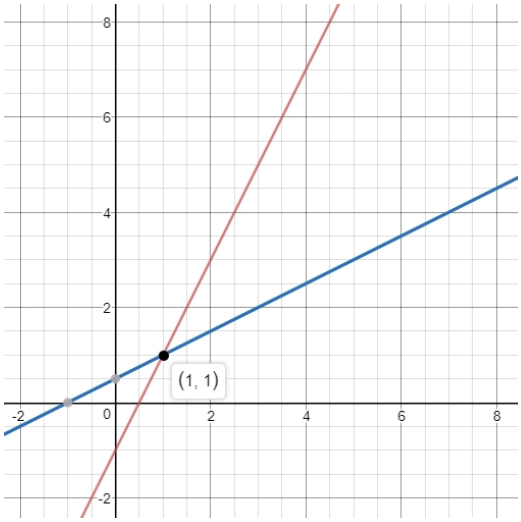
$2x - 1 = \frac{x+1}{2}$

$4x - 2 = x + 1$

$\frac{3x}{3} = \frac{3}{3}$

$x = 1$

$\Rightarrow (1, 1)$



Slopes are the reciprocal of each other (2 vs $\frac{1}{2}$)

9. a) Determine f^{-1} for the linear function $f(x) = 5x - 2$.
b) Graph f and f^{-1} on the same axes.
c) Explain how you can tell that f^{-1} is also a linear function.
d) State the coordinates of any points that are common to both f and f^{-1}.
e) Compare the slopes of the two lines.
f) Repeat parts (a) to (e) for $g(x) = -\frac{1}{2}x + 3$, $h(x) = 2x - 1$, $p(x) = 6 - x$, and $q(x) = 2$.

$y = 6 - x$

$x = 6 - y$

$x + y = 6$

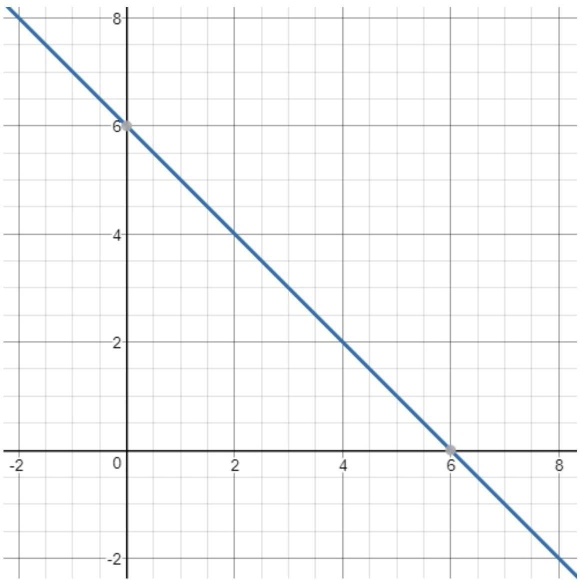
$y = 6 - x$

$p^{-1}(x) = 6 - x$

Straight line graph

All points are on both.

They are SELF INVERSES.



Slopes are reciprocal of each other (-1 vs -1)

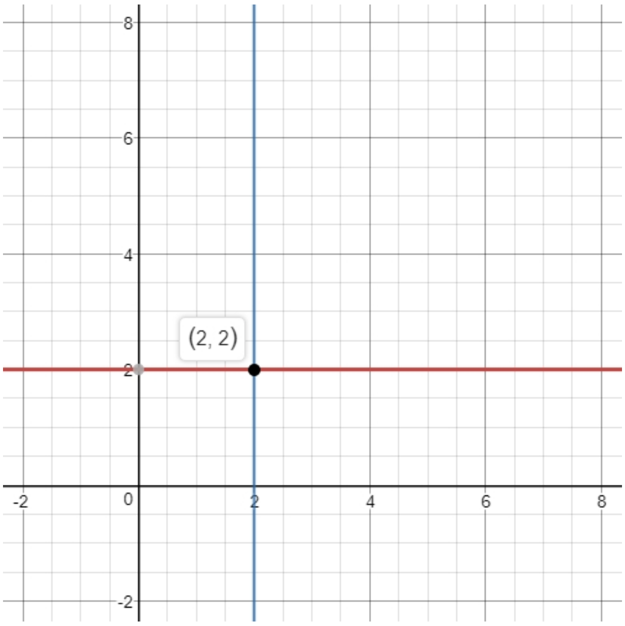
9. a) Determine f^{-1} for the linear function $f(x) = 5x - 2$.
b) Graph f and f^{-1} on the same axes.
c) Explain how you can tell that f^{-1} is also a linear function.
d) State the coordinates of any points that are common to both f and f^{-1}.
e) Compare the slopes of the two lines.
f) Repeat parts (a) to (e) for $g(x) = -\frac{1}{2}x + 3$, $h(x) = 2x - 1$, $p(x) = 6 - x$, and $q(x) = 2$.

$y = 2$
$y = 0x + 2$
$x = 0y + 2$
$x = 2$
$g^{-1}(x): x = 2$

Straight line, but NOT a function

Common point of $(2,2)$

Slopes are 0 and undefined



10. For $g(t) = 3t - 2$, determine each value.

a) $g(13)$
b) $g(7)$
c) $\dfrac{g(13) - g(7)}{13 - 7}$
d) $g^{-1}(13)$
e) $g^{-1}(7)$
f) $\dfrac{g^{-1}(13) - g^{-1}(7)}{13 - 7}$

$y = 3t - 2$
$\dfrac{y+2}{3} = \dfrac{3t}{3}$ $\Rightarrow g^{-1}(t) = \dfrac{t+2}{3}$

a) $g(13) = 3(13) - 2$
$= 39 - 2$
$= 37$

b) $g(7) = 3(7) - 2$
$= 21 - 2$
$= 19$

c) $= \dfrac{37 - 19}{13 - 7}$
$= \dfrac{18}{6}$
$= 3$

d) $g^{-1}(13) = \dfrac{13+2}{3}$
$= \dfrac{15}{3}$
$= 5$

e) $g^{-1}(7) = \dfrac{7+2}{3}$
$= \dfrac{9}{3}$
$= 3$

f) $= \dfrac{5-3}{13-7}$
$= \dfrac{2}{6}$
$= \dfrac{1}{3}$

17. Given $f(x) = k(2 + x)$, find the value of k if $f^{-1}(-2) = -3$.

T

$$y = k(2+x)$$

$$\frac{x}{k} = \frac{k(2+y)}{k}$$

$$\frac{x}{k} = 2+y$$

$$\frac{x}{k} - 2 = y$$

$$f^{-1}(x) = \frac{x}{k} - 2$$

$$f^{-1}(-2) = -3$$

$$\frac{-2}{k} - 2 = -3$$

$$\frac{-2}{k} = -1$$

$$\frac{-2}{-1} = \frac{-k}{-1}$$

$$2 = k$$