

# Solutions

1. Convert to factorial form, then evaluate.

a)  ${}_9C_5$

b)  ${}_8C_4$

c)  $C(12,3)$

d)  $\binom{11}{5}$

e)  ${}_7C_2 \times {}_6C_3$

f)  $\binom{101}{98} \times \binom{101}{3}$

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

$$\begin{aligned} \text{a)} \quad & \frac{9!}{(9-5)!5!} \\ & = \frac{9!}{4!5!} \\ & = 126 \end{aligned}$$

$$\begin{aligned} \text{b)} \quad & \frac{8!}{(8-4)!4!} \\ & = \frac{8!}{4!4!} \\ & = 70 \end{aligned}$$

$$\begin{aligned} \text{c)} \quad & \frac{12!}{(12-3)!3!} \\ & = \frac{12!}{9!3!} \\ & = 220 \end{aligned}$$

$$\begin{aligned} \text{d)} \quad & \frac{11!}{(11-5)!5!} \\ & = \frac{11!}{6!5!} \\ & = 462 \end{aligned}$$

$$\begin{aligned} \text{e)} \quad & \frac{7!}{(7-2)!2!} \times \frac{6!}{(6-3)!3!} \\ & = \frac{7!}{5!2!} \times \frac{6!}{3!3!} \\ & = 21 \times 20 \\ & = 420 \end{aligned}$$

$$\begin{aligned} \text{f)} \quad & \frac{101!}{(101-98)!98!} \times \frac{101!}{(101-3)!3!} \\ & = \frac{101!}{3!98!} \times \frac{101!}{98!3!} \\ & = 166,650 \times 166,650 \\ & = 2.777225 \times 10^{10} \end{aligned}$$

2. Which is an incorrect way of writing  ${}_{10}C_3$ ?

A  $\frac{{}_{10}P_3}{3!}$

B  $\frac{{}_{10}P_3}{7!}$

C  $\frac{10!}{7!3!}$

D  $\frac{10!}{3!7!}$

$$\begin{aligned} {}_{10}C_3 &= \frac{10!}{(10-3)!3!} \\ &= \frac{10!}{7!3!} \end{aligned}$$

Remember

$${}_nC_r = \frac{{}_nP_r}{r!}$$

So...

$${}_{10}C_3 = \frac{{}_{10}P_3}{3!}$$

$\Rightarrow$  B is incorrect

3. How many three-member committees can be formed from a group of nine people?

A 27

B 84

C 504

D 729

9 members

3 to be chosen

Non specific roles  $\Rightarrow$  order is NOT important

$${}_9C_3 = \frac{9!}{(9-3)!3!}$$

$$= \frac{9!}{6!3!}$$

$$= 84 \text{ ways} \Rightarrow \text{B}$$

4. In how many ways could 6 online magazine subscriptions be chosen from a set of 10 magazines?

Order is NOT important

$$\begin{aligned}\Rightarrow 10C_6 &= \frac{10!}{(10-6)!6!} \\ &= \frac{10!}{4!6!} \\ &= 210 \text{ ways}\end{aligned}$$

5. In how many ways could you choose 4 packages of pasta from a bin containing 11 different packages of pasta?

$$\begin{aligned}11C_4 &= \frac{11!}{(11-4)!4!} \\ &= \frac{11!}{7!4!} \\ &= 330 \text{ ways}\end{aligned}$$

8. **Application** On an English exam, students need to answer six out of eight questions in Part A and two out of four questions in Part B. The order in which they answer the questions does not matter. In how many ways could a student answer the questions on this exam?

Answer 6 out of 8  
AND 2 out of 4

$$\begin{aligned} \Rightarrow & 8C_6 \times 4C_2 \\ &= \frac{8!}{(8-6)!6!} \times \frac{4!}{(4-2)!2!} \\ &= \frac{8!}{2!6!} \times \frac{4!}{2!2!} \\ &= 28 \times 6 \\ &= 168 \text{ ways} \end{aligned}$$

10. **Communication** Juries are chosen from large pools of people selected at random from the local population. A jury pool has 40 people.

a) How many ways are there to form a 12-person jury in a criminal case?

b) How many ways are there to form a 6-person jury in a civil case?

c) Which situation gives a larger number of ways? Explain why this is to be expected.

a) 12 from 40

$$\begin{aligned} \Rightarrow 40C_{12} &= \frac{40!}{(40-12)!12!} \\ &= \frac{40!}{28!12!} \\ &= 5,586,853,480 \end{aligned}$$

b) 6 from 40

$$\begin{aligned} \Rightarrow 40C_6 &= \frac{40!}{(40-6)!6!} \\ &= \frac{40!}{34!6!} \\ &= 3,838,380 \end{aligned}$$

c) 12 from 40 gives the larger number of ways. This is because you are dividing by a smaller number [28!6! < 34!6!]