Combinations

Lesson objectives

- I can recognise the advantages of using permutations and combinations over other counting techniques
- I can apply combinations to solve counting problems
- I can express combinations in standard notation:

C(n,r) nCr $\binom{n}{r}$

1.1

Lesson objectives | Teachers' notes | Lesson notes

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Warm Up

You have a toonie, loonie, quarter and a dime. How many different sums of money can you make from the coins?

$_4P_1 + {}_4P_2 + {}_4P_3 + {}_4P_4$

However the order is NOT important.

Use 1 coin

2 , 1 , 0.25, 0.10

Use 2 coins

2 + 1 = 3 1 + 0.25 = 1.25
2 + 0.25 = 2.25 1 + 0.1 = 1.10
2 + 0.10 = 2.10 0.25 + 0.1 = 0.35

Use 3 coins

2 + 1 + 0.25 = 3.25 2 + 0.25 + 0.10 = 2.35
2 + 1 + 0.10 = 3.10 1 + 0.25 + 0.10 = 1.35

Use 4 coins

2 + 1 + 0.25 + 0.10 = 3.35

Total of 15 ways.

Definition

Combination

- A selection from a group of objects without regard to the order.

Combinations

In the previous section, you divided the permutation formula by the number of ways of arranging the identical items to account for identical items. Similarly, when choosing r items from a set of n items, without regard to order, divide the permutation formula by $r!$.

The number of combinations of r objects chosen from a set of n items is

$$_nC_r = \frac{_nP_r}{r!} = \frac{\frac{n!}{(n-r)!}}{r!} = \frac{n!}{(n-r)!r!}$$

Other standard combination notation is $\binom{n}{r}$ and $C(n, r)$. This is read as "n choose r."

Example 1

Choose Items From a Set

How many ways can a five-card hand be dealt from a standard deck?

The order you get the cards in is not important.

$$_{52}C_5 = \frac{52!}{(52-5)!5!}$$

$$= 2,598,960$$

Your Turn

In a competition, junior chefs make a gourmet soup by selecting from 10 different ingredients. How many different soups can the chefs make if the soup must include

a) four of the ingredients?

$$_{10}C_4 = \frac{10!}{(10-4)!4!} = 210$$

b) five of the ingredients?

$$_{10}C_5 = \frac{10!}{(10-5)!5!} = 252$$

c) six of the ingredients?

$$_{10}C_6 = \frac{10!}{(10-6)!6!} = 210$$

Example 2

Choose More Than One Group

A committee of 3 men and 3 women is formed from a group of 8 men and 10 women. How many ways are there to form the committee?

$n(\text{Men}) = {}_8C_3 = 56$

$n(\text{Women}) = {}_{10}C_3 = 120$

$\Rightarrow n(\text{men AND women}) = {}_8C_3 \times {}_{10}C_3$

$= 56 \times 120$

$= 6720$

Your Turn

Erica is making a platter of four types of cheese and four types of crackers. She has seven different cheeses and six different crackers. In how many ways can Erica make the platter?

$n(\text{cheese}) = {}_7C_4$

$n(\text{crackers}) = {}_6C_4$

$\Rightarrow n(\text{cheese AND crackers}) = {}_7C_4 \times {}_6C_4$

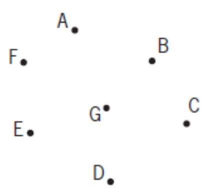
$= 35 \times 15$

$= 525$

Example 3

Interpret a Diagram

The seven points represent cabins at a lodge. How many paths can be drawn by joining pairs of cabins?



Need 2 letters to make a path. Order is not important, as path AB is identical to path BA.

$\Rightarrow n(\text{paths}) = {}_7C_2 \leftarrow$ choose 2 points from 7

$= 21$

Your Turn

How many triangles can be drawn using the seven points as vertices?

Need 3 vertices for a triangle.

$\Rightarrow n(\text{triangles}) = {}_7C_3 \leftarrow$ choose 3 vertices from 7

$= 35$

Key Concepts

- A combination is a set of items taken from another set in which order does not matter. In a permutation, the order of the items matters.
- The number of combinations of r items taken from a set of n items is $_nC_r = \frac{n!}{(n-r)!\,r!}$.

R3. Which situation has a greater number of possibilities, one in which order matters or one in which order does not matter? Explain why.

A situation in which order matters (permutations) will have more possibilities.

By definition $_nC_r = \frac{_nP_r}{r!}$ For each combination of r times there are r! permutations. So, the number of combinations is r! times smaller than the number of permutations.

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