Solutions

1. Simplify.

a) $\frac{10!}{2!3!5!}$

b) $\frac{9!}{3!3!3!}$

c) $\frac{7!}{2!3!}$

d) $\frac{120!}{115!3!2!}$

a) $\frac{10 \times 9 \times \cancel{8}^{4} \times 7 \times \cancel{6} \times \cancel{5!}}{\cancel{2} \times 1 \times \cancel{3 \times 2} \times 1 \times \cancel{5!}}$

$= 10 \times 9 \times 4 \times 7$

$= 2520$

b) $\frac{\cancel{9}^{3} \times \cancel{8}^{4} \times 7 \times \cancel{6} \times 5 \times 4 \times \cancel{3!}}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{3 \times 2} \times 1 \times \cancel{3!}}$

$= 3 \times 4 \times 7 \times 5 \times 4$

$= 1680$

c) $\frac{7 \times \cancel{6}^{3} \times 5 \times 4 \times \cancel{3!}}{\cancel{2} \times 1 \times \cancel{3!}}$

$= 7 \times 3 \times 5 \times 4$

$= 420$

d) $\frac{\cancel{120}^{10} \times 119 \times 118 \times 117 \times 116 \times \cancel{115!}}{\cancel{115!} \times \cancel{3 \times 2 \times 1 \times 2} \times 1}$

$= 10 \times 119 \times 118 \times 117 \times 116$

$= 1,905,780,240$

2. What is the number of arrangements of five small tiles and three large tiles?

A 20

B 56

C 720

D 40 320

5 large, 3 small, 8 in total

$$n(A) = \frac{n!}{p!q!r!\ldots}$$

$$= \frac{8!}{5!3!}$$

$$= \frac{8 \times 7 \times \cancel{6} \times \cancel{5!}}{\cancel{5!} \times \cancel{3 \times 2} \times 1}$$

$$= 8 \times 7$$

$$= 56 \implies B$$

3. Dana has 12 pens. There are four blue, three red, and the others are different colours. Which set of values for the variables in a permutation calculation is correct?

A $n = 12, p = 4, q = 3, r = 5$

B $n = 7, p = 4, q = 3$

C $n = 12, p = 4, q = 3, r = 0$

D $n = 12, p = 4, q = 3$

$$n(A) = \frac{n!}{p!q!r!\ldots}$$

12 pens
4 blue
3 red

$\implies n = 12$
$p = 4$
$q = 3$

No other colour is repeated, so we don't need "r" [really r = 1]

$\implies D$

4. How many permutations are there of all the letters in each name?

a) WATERLOO

b) TORONTO

c) MISSISSAUGA

d) OTTAWA

a) 8 letters
2 "O's"
$\Rightarrow \frac{8!}{2!} = 20160$

b) 7 letters
2 "T's", 3 "O's"
$\Rightarrow \frac{7!}{2!3!} = 420$

c) 11 letters
2 "I's", 4 "S's", 2 "A's"
$\Rightarrow \frac{11!}{2!4!2!} = 415{,}800$

d) 6 letters
2 "T's", 2 "A's"
$\Rightarrow \frac{6!}{2!2!} = 180$

5. How many five-digit numbers can be formed using each set of numbers?

a) 1, 2, 2, 3, 4

b) 1, 2, 2, 2, 3

c) 1, 1, 2, 3, 3

d) 1, 2, 2, 2, 2

a) 5 numbers
2 "2s"
$\Rightarrow \frac{5!}{2!} = 60$

b) 5 numbers
3 "2s"
$\Rightarrow \frac{5!}{3!} = 20$

c) 5 numbers
2 "1s", 2 "3s"
$\Rightarrow \frac{5!}{2!2!} = 30$

d) 5 numbers
4 "2s"
$\Rightarrow \frac{5!}{4!} = 5$

6. Sam has four different types of fruit. He has three pieces of each type. In how many ways could he arrange them on a platter
   a) in a line?
   b) in three rows of four?
   c) in two rows of six?

3 pieces of 4 different fruits = 12 in total

a) $\frac{12!}{3!3!3!3!} = 369,600$

arranging the 3 pieces of each fruit

b) $\frac{12!}{4!4!4!} = 34,650$

arranging each row of 4

c) $\frac{12!}{6!6!}$

arranging each row of 6

$= 924$

7. In a panel of eight light switches, half are on and half are off. In how many ways could this be done?

8 switches
4 on
4 off

$n(A) = \frac{8!}{4!4!} = 70$

arrangements of "ON"

arrangements of "OFF"

8. In one of her tricks, a clown rearranges two identical quarters, three identical loonies, and five identical toonies in a row. In how many ways can the clown arrange the coins?

2 quarters
3 loonies
5 toonies
Total of 10 coins

$$\Rightarrow n(A) = \frac{10!}{2!\,3!\,5!} = 2520$$

arrangements of quarters
" loonies
" toonies

9. How many arrangements are there of 15 flags in a row if five are red, four are green, two are blue, and four are yellow?

5 red, 4 green, 2 blue, 4 yellow

$$\Rightarrow n(A) = \frac{15!}{5!\,4!\,2!\,4!} = 9{,}459{,}450$$

arrangements of red
" green
" blue
" yellow

13. In how many ways could the letters in the word PROBLEM be arranged if the consonants must remain in the original order?

Treat the consonants as being the same letter because their order can't change.

Let C = consonant

$\Rightarrow$ PROBLEM = CCOCCEC

$$n(A) = \frac{7!}{5!} = 42$$