Investigation of Inverse Functions

Nelson Page 41

Complete the investigation answering parts A through to O.

The Backyard Paving Company charges \$10/sq ft for installing interlocking paving stones, plus a \$50 delivery fee. The company calculates the cost to the customer as a function of the area to be paved. Tom wants to express area in terms of cost to see how much of his yard he can pave for different budget amounts.

A. Copy and complete table A, using the company's prices. What is the independent variable in table A? the dependent variable?

Independent → Area (x)

Dependedependent → Cost (y)

A

x Area (sq ft)	*y* Cost ($)
40	450
80	850
120	1250
160	1650
200	2050

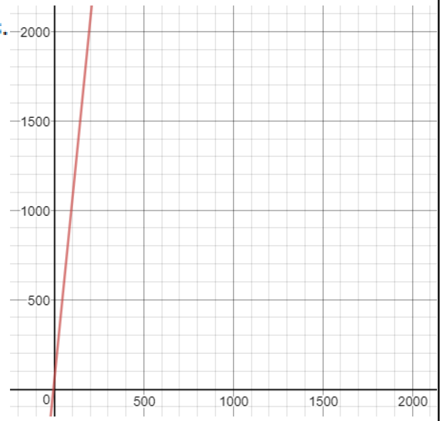
B. Is the relation in table A a function? Explain.

Yes. Each area has only one cost.

C. Write the equation for $f(x)$ that describes the cost as a function of area.

$f(x) = 10x + 50$

D. Graph $f(x)$. Use the same scale of -100 to 2100 on each axis.



E. Tom needs to do the reverse of what the company's function does. Copy and complete table E for Tom. What is the independent variable? the dependent variable? How does this table compare with table A?

Independent → Cost

Dependepent → Area

It is reversed compared to Table A

E

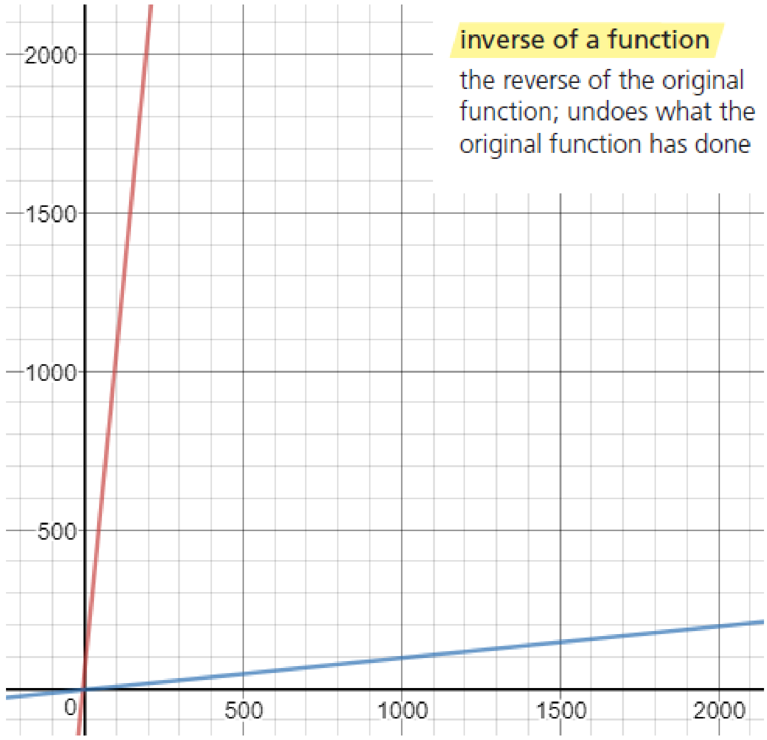
Cost ($)	Area (sq ft)
450	40
850	80
1250	120
1650	160
2050	200

F. The relationship in part E is the inverse of the cost function. Graph this inverse relation on the same axes as those in part D. Is this relation a function? Explain.

Yes.
Each cost only has one area.

inverse of a function

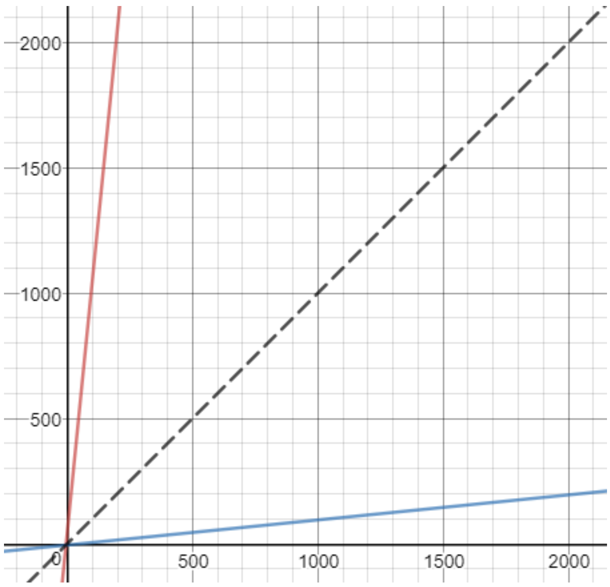
the reverse of the original function; undoes what the original function has done



G. Draw the line $y = x$ on your graph. Place a Mira along the line $y = x$, or fold your graph paper along that line. What do you notice about the two graphs? Where do they intersect?

They reflect in the line $y = x$.

Intersect at $\left(-5\frac{5}{9}, -5\frac{5}{9}\right)$



H. Compare the coordinates of points that lie on the graph of the cost function with those which lie on the graph of its inverse. What do you notice?

They are the reverse of each other.

I. Write the slopes and y-intercepts of the two lines.

i) How are the slopes related?

ii) How are the y-intercepts related?

iii) Use the slope and y-intercept to write an equation for the inverse.

	Slope	y-int
A	10	50
B	$\frac{1}{10}$	-5

i) Reciprocal of each other

ii) A: slope m, y-int b

B: slope $\frac{1}{m}$, y-int $\frac{-b}{m}$

iii) $y = \frac{x}{10} - 5$

J. Use inverse operations on the cost function, f, to solve for x. Compare this equation with the equation of the inverse you found in part I.

$f(x) = 10x + 50$

$f(x) - 50 = 10x$

$\frac{f(x) - 50}{10} = x$

$\frac{f(x)}{10} - 5 = x$

They are the same!

K. Make a list of all the connections you have observed between the Backyard Paving Company's cost function and the one Tom will use.

Input and output will be reversed

Graphs reflect in the line $y=x$

L. How would a table of values for a linear function help you determine the inverse of that function?

Reverse the columns and then look to create the equation.

M. i) How can you determine the coordinates of a point on the graph of the inverse function if you know a point on the graph of the original function?

ii) How could you use this relationship to graph the inverse?

(i) Switch the x and y values

(ii) Plot the new points and join them up

N. How are the domain and range of the inverse related to the domain and range of a linear function?

Domain becomes the range
Range becomes the domain

O. How could you use inverse operations to determine the equation of the inverse of a linear function from the equation of the function?

Interchange the x and y values.
Solve for x.
This is the inverse function.