Unit 2 Functions Review

Page 76 #s 1 - 8 & 12 -17

Nov 20-18:34

Solutions

Nov 20-18:35

1. For each relation, determine the domain and range and whether the relation is a function. Explain your reasoning.

a) $\{(-3, 0), (-1, 1), (0, 1), (4, 5), (0, 6)\}$

$D = \{-3, -1, 0, 4\}$

$R = \{0, 1, 5, 6\}$

Not a function because $x = 0$ has 2 values

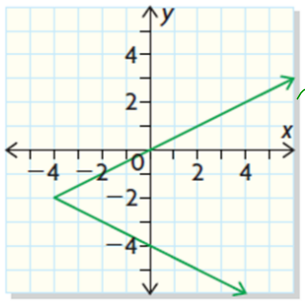
b) $y = 4 - x$

$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R}\}$

Function each x-value has only one y-value.

c)



Not a function fails the VLT

$D = \{x \in \mathbb{R} \mid x \geq -4\}$

$R = \{y \in \mathbb{R}\}$

d) $x^2 + y^2 = 16$

Jan 22-09:46

2. What rule can you use to determine, from the graph of a relation, whether the relation is a function? Graph each relation and determine which are functions.

a) $\{(-2. 1), (1, 1), (0, 0), (1, -1), (1, -2), (2, -2)\}$

b) $y = 4 - 3x$

c) $y = (x - 2)^2 + 4$

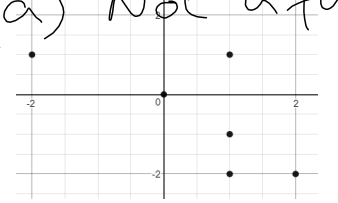
d) $x^2 + y^2 = 1$

e) $y = \frac{1}{x}$

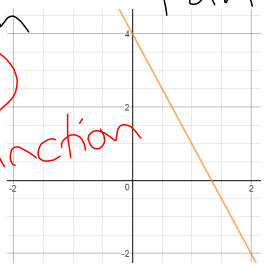
f) $y = \sqrt{x}$

The Vertical Line Test (VLT). If a vertical line crosses the relation more than once then it is not a function.

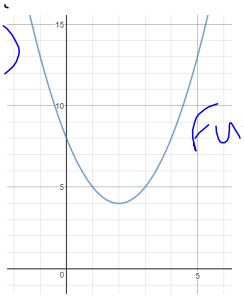
(a) Not a function



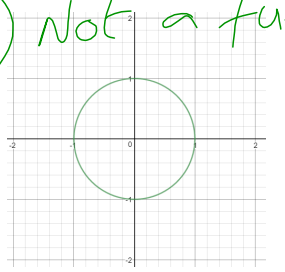
(b) Function



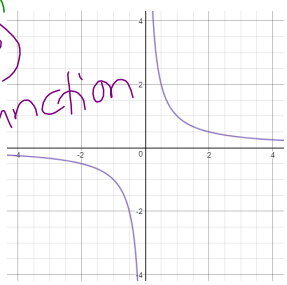
(c) Function



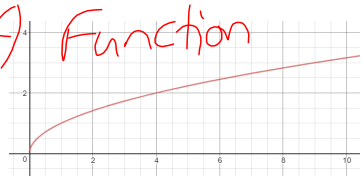
(d) Not a function



(e) Function

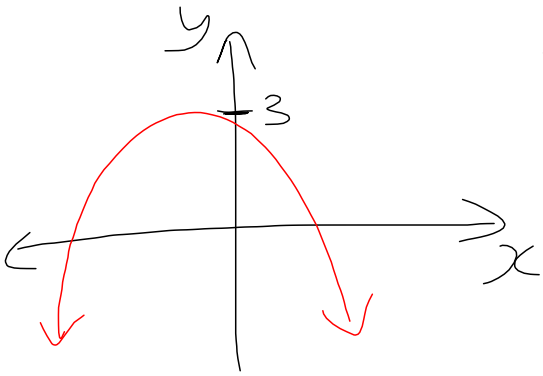


(f) Function



Jan 22-09:46

3. Sketch the graph of a function whose domain is the set of real numbers and whose range is the set of real numbers less than or equal to 3.



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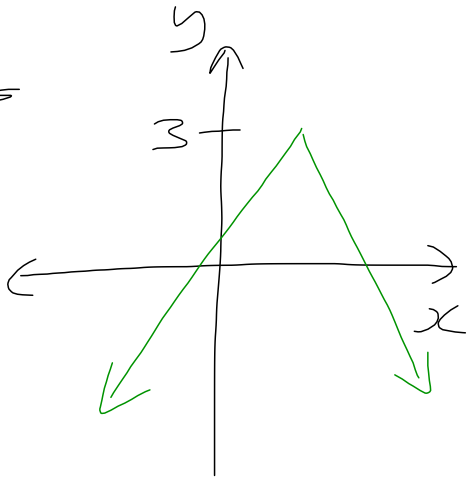


Jan 22-09:46

4. If $f(x) = x^2 + 3x - 5$ and $g(x) = 2x - 3$, determine each.

a) $f(-1)$

$= (-1)^2 + 3(-1) - 5$

$= 1 - 3 - 5$

$= -7$

d) $f(2b)$

$= (2b)^2 + 3(2b) - 5$

$= 4b^2 + 6b - 5$

c) $g\left(\frac{1}{2}\right)$

$= 2\left(\frac{1}{2}\right) - 3$

$= 1 - 3$

$= -2$

b) $f(0)$

$= (0)^2 + 3(0) - 5$

$= 0 + 0 - 5$

$= -5$

e) $g(1 - 4a)$

$= 2(1 - 4a) - 3$

$= 2 - 8a - 3$

$= -8a - 1$

f) x when $f(x) = g(x)$

$x^2 + 3x - 5 = 2x - 3$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2, x = 1$

Jan 22-09:46

5. a) Graph the function $f(x) = -2(x-3)^2 + 4$, and state its domain and range.

b) What does $f(1)$ represent on the graph? Indicate, on the graph, how you would find $f(1)$.

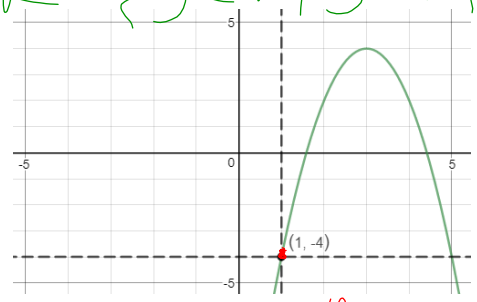
c) Use the equation to determine each of the following.

i) $f(3) - f(2)$ iii) $f(1-x)$

ii) $2f(5) + 7$

$D = \{x \in R\}$

$R = \{y \in R \mid y \leq 4\}$



b) Represents the y-value when $x = 1$.

i) $f(3) = -2(3-3)^2 + 4$
$= 4$
$f(2) = -2(2-3)^2 + 4$
$= 2$
$\Rightarrow f(3) - f(2) = 2$

ii) $2f(5) = 2[-2(5-3)^2 + 4]$
$= 2[-4] = -8$
$\Rightarrow 2f(5) + 7 = -1$

iii) $f(1-x) = -2((1-x)-3)^2 + 4$
$= -2(-2-x)^2 + 4$
$= -2(x^2 + 4x + 4) + 4$
$= -2x^2 - 8x - 8 + 4$
$= -2x^2 - 8x - 4$

Jan 22-09:46

6. If $f(x) = x^2 - 4x + 3$, determine the input(s) for x whose output is $f(x) = 8$.

$8 = x^2 - 4x + 3$

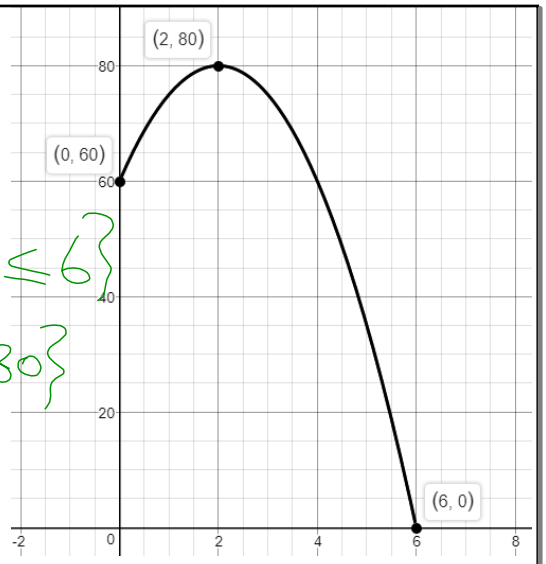
$0 = x^2 - 4x - 5$

$0 = (x-5)(x+1)$

$x = 5, \; x = -1$

Jan 22-09:46

7. A ball is thrown upward from the roof of a building 60 m tall. The ball reaches a height of 80 m above the ground after 2 s and hits the ground 6 s after being thrown.
   a) Sketch a graph that shows the height of the ball as a function of time.
   b) State the domain and range of the function.
   c) Determine an equation for the function.



b) $D = \{x \in R \mid 0 \le x \le 6\}$

$R = \{y \in R \mid 0 \le y \le 80\}$

c) $y = a(x-h)^2 + k$

$(h,k) = (2,80)$

$(x,y) = (0,60)$

Sub in and solve for "a"

$60 = a(0-2)^2 + 80$

$\frac{-20}{4} = \frac{a(4)}{4}$ $a = -5$

$\Rightarrow y = -5(x-2)^2 + 80$

Jan 22-09:46

8. State the domain and range of each function.
   a) $f(x) = 2(x-1)^2 + 3$
   b) $f(x) = \sqrt{2x+4}$

a) $D = \{x \in R\}$

$R = \{y \in R \mid y \ge 3\}$

b) Factor out "k"

$= \sqrt{2(x+2)}$

$D = \{x \in R \mid x \ge -2\}$

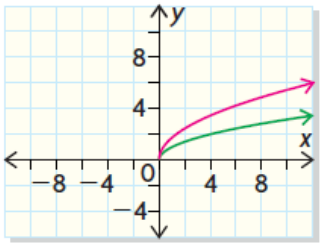
$R = \{y \in R \mid y \ge 0\}$

Jan 22-09:46

12. In each graph, a parent function has undergone a transformation of the form $f(kx)$. Determine the equations of the transformed functions graphed in red. Explain your reasoning.

ORIGINAL = GREEN
TRANSFORMED = PURPLE

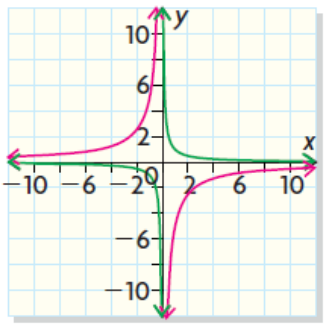
a)



Horizontally Compressed by a factor of $\frac{1}{4}$ $(4,1) \Rightarrow (1,1)$

$\Rightarrow k = \frac{1}{\frac{1}{4}} = 4 \Rightarrow y = 4x$

b)



Reflected in the y-axis
Horizontally stretched factor 5
$(1,1) \rightarrow (-5,1)$

$\Rightarrow k = \frac{1}{-5} \Rightarrow y = \frac{1}{-5x}$

Jan 22-09:46

13. For each set of functions, transform the graph of $f(x)$ to sketch $g(x)$ and $h(x)$, and state the domain and range of each function.

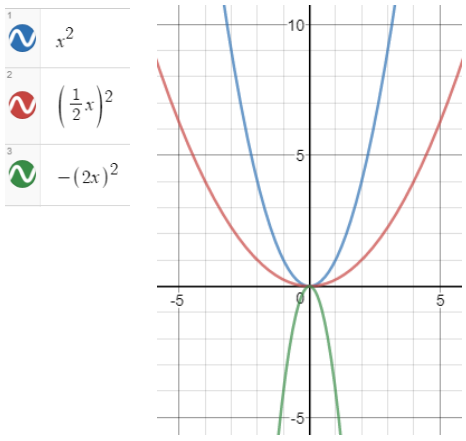
All domains $D = \{x \in \mathbb{R}\}$

a) $f(x) = x^2, g(x) = \left(\frac{1}{2}x\right)^2, h(x) = -(2x)^2$

$R = \{y \in \mathbb{R} \mid y \geq 0\}$

$R = \{y \in \mathbb{R} \mid y \geq 0\}$

$R = \{y \in \mathbb{R} \mid y \leq 0\}$



b) $f(x) = |x|, g(x) = |-4x|, h(x) = \left|\frac{1}{4}x\right|$

$R = \{y \in \mathbb{R} \mid y \geq 0\}$

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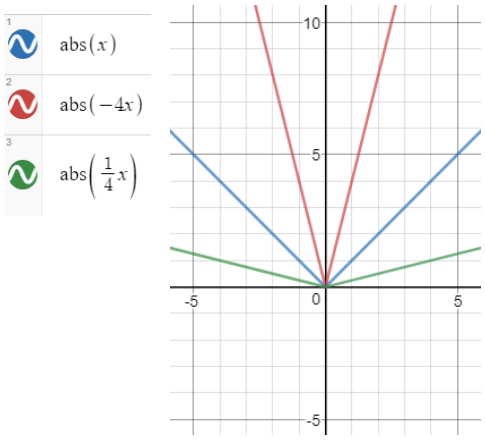


Jan 22-09:46

14. Three transformations are applied to $y = x^2$: a vertical stretch by the factor 2, a translation 3 units right, and a translation 4 units down.

a) Is the order of the transformations important?

b) Is there any other sequence of these transformations that could produce the same result?

a) Yes. Stretch before you move.

b) Translations can be performed in either order.

Jan 22-09:46

15. The point $(1, 4)$ is on the graph of $y = f(x)$. Determine the coordinates of the image of this point on the graph of $y = 3f[-4(x + 1)] - 2$.

$a = 3 \quad k = -4 \quad d = -1 \quad c = -2$

$x \rightarrow \frac{x}{k} + d \qquad 1 \rightarrow \frac{1}{-4} + (-1) = -1\frac{1}{4}$

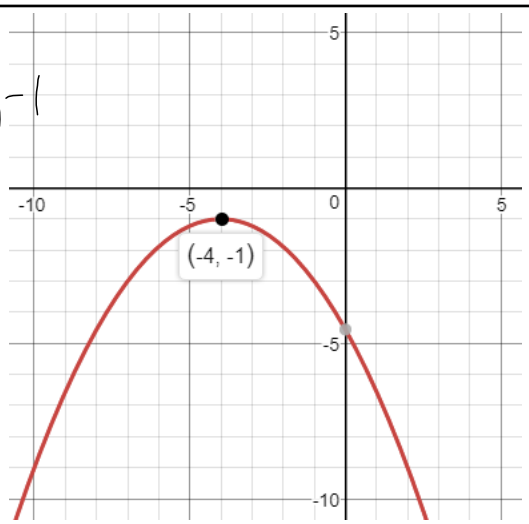
$y \rightarrow ay + c \qquad 4 \rightarrow 3(4) - 2 = 10$

The point $(1, 4)$ moves to $(-1\frac{1}{4}, 10)$

Jan 22-09:46

16. a) Explain what you would need to do to the graph of $y = f(x)$ to graph the function $y = -2f\left[\frac{1}{3}(x+4)\right] - 1$

b) Graph the function in part (a) for $f(x) = x^2$.



Reflect in the x-axis

VS factor of 2

HS factor of 3

HT left 4

VT down 1

Jan 22-09:46

17. In each case, write the equation for the transformed function, sketch its graph, and state its domain and range.

a) The graph of $f(x) = \sqrt{x}$ is compressed horizontally by the factor $\frac{1}{2}$, reflected in the y-axis, and translated 3 units right and 2 units down.

$k = \frac{1}{1/2} = 2$

k is negative

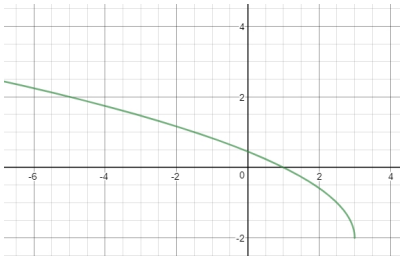
$d = 3$

$c = -2$

$\Rightarrow f(x) = \sqrt{-2(x-3)} - 2$

$D = \{x \in R \mid x \leq 3\}$

$R = \{y \in R \mid y \geq -2\}$



b) The graph of $y = \frac{1}{x}$ is stretched vertically by the factor 3, reflected in the x-axis, and translated 4 units left and 1 unit up.

$a = 3$

a is negative

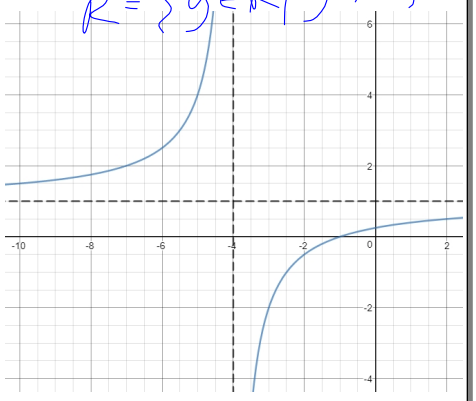
$d = -4$

$c = 1$

$\Rightarrow f(x) = \frac{-3}{x+4} + 1$

$D = \{x \in R \mid x \neq -4\}$

$R = \{y \in R \mid y \neq 1\}$



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