Permutations

Extra Practice

MHR Page 98 #s 1 - 13

Solutions

1. How many orders of faces are possible when a standard die is rolled four times?

A 16
B 24
C 1296
D 4096

6 outcomes for each roll

$\Rightarrow 6 \times 6 \times 6 \times 6$

$= 6^4$

$= 1296 \Rightarrow C$

2. Which of the following is equivalent to ${}_{101}P_{98}$?

A 3!
B $101 \times 100 \times 99 \times 98$
C $\frac{101!}{98!}$
D $\frac{101!}{3!}$

$nP_r = \frac{n!}{(n-r)!}$

$\Rightarrow {}_{101}P_{98} = \frac{101!}{(101-98)!}$

$= \frac{101!}{3!} \Rightarrow D$

3. When flipping a coin five times, what is the probability that heads turns up every time?

A $\frac{1}{32}$

B $\frac{5}{32}$

C $\frac{1}{10}$

D $\frac{1}{25}$

2 outcomes when flipping a coin

$P(\text{Head}) = \frac{1}{2}$

$P(5 \text{ Heads}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{2^5}$

$= \frac{1}{32} \Rightarrow A$

4. Which of the following is not defined? Explain your reasoning.

- ${}_{12}P_8$
- ${}_{9}P_{10}$
- ${}_{7}P_{0}$
- ${}_{100}P_{100}$

${}_9P_{10}$ is undefined

You cannot choose to arrange 10 items from a total of 9 items.

Also $\frac{9!}{(9-10)!} = \frac{9!}{(-1)!}$

$(-1)!$ is undefined

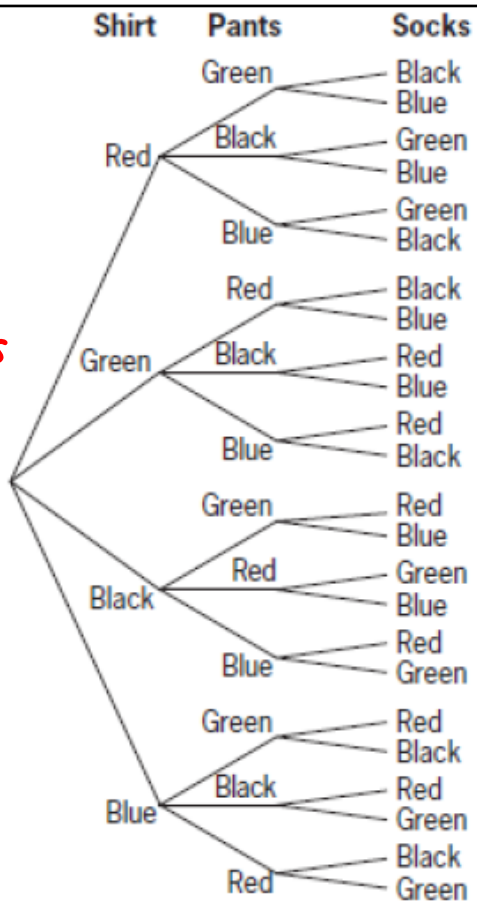
5. Rosa is getting dressed and has decided that her shirt, pants, and socks are not to be the same colour. She has red, green, black, and blue of each.

a) Draw a tree diagram illustrating her choices.

b) How many choices does she have if she starts with a red pair of pants?

If she starts with red pants she has 3 choices for shirt/socks and then 2 choices for socks/shirt

$\Rightarrow 3 \times 2 = 6$ choices



6. A hockey team has four left wingers, three right wingers, four centres, three left defence, four right defence, and two goalies. To create a starting lineup, a coach needs one player in each position. In how many ways could the starting lineup be chosen?

4 LW, 3 RW, 4 C, 3 LD, 4 RD, 2 G

# of lineups $= 4 \times 3 \times 4 \times 3 \times 4 \times 2$

$= 1152$

7. How many ways are there to assign five different roles in a play to the 12 members of a drama club?

Choosing 5 from 12

$\Rightarrow {}_{12}P_5 = \frac{12!}{(12-5)!}$

$= \frac{12!}{7!}$ $(12 \times 11 \times 10 \times 9 \times 8)$

$= 95,040$

8. There are three Canadians in the finals at a ski competition. Assuming all eight competitors are equally likely to win, what is the probability that the three Canadians will win gold, silver, and bronze?

$P(\text{Gold}) = \frac{3}{8}$

$P(\text{Silver}) = \frac{2}{7}$

$P(\text{Bronze}) = \frac{1}{6}$

$\Rightarrow P(\text{Gold, Silver \& Bronze}) = P(G) \times P(S) \times P(B)$

$= \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6}$

$= \frac{6}{336}$

$= \frac{1}{56}$

9. a) How many arrangements are there of the letters in the word COMPUTER?

b) How many of them begin with a consonant?

a) 8 letters $\Rightarrow$ 8! arrangements
$= 40,320$

b) 5 consonants to choose from
$= 5 \times 7!$
$= 25,200$

OR
$5P_1 \times 7P_7$
$= 5 \times 5040$
$= 25,200$

10. In how many ways could the 11 members of a soccer team line up if the captain and assistant captain must remain apart?

Treat C and AC as one player

$\Rightarrow$ # of arrangements with them together $= 10! \times 2!$

$10!$: other players; $2!$: C and AC

Total # of arrangements $= 11!$

$\Rightarrow$ # of arrangements with C and AC not together $= 11! - (10! \times 2!)$
$= 39,916,800 - 7,257,600$
$= 32,659,200$

11. There are 25 men and 20 women who belong to a club. An executive panel consisting of a president, vice president, secretary, and treasurer is being chosen.

a) In how many ways could the executive panel be chosen with no restrictions?

b) In how many ways could the executive panel be chosen if it must include at least one woman and one man?

c) In how many ways could the executive panel be chosen if the president and vice president must have different genders?

25 men, 20 women, 45 people in total

a) $45 \times 44 \times 43 \times 42$

$= 3,575,880$ ways

$[{}_{45}P_4]$

b) # of ways no women

$= 25 \times 24 \times 23 \times 22$

$= 303,600$ $({}_{25}P_4)$

# of ways no men

$= 20 \times 19 \times 18 \times 17$

$= 116,280$ $({}_{20}P_4)$

$\Rightarrow$ # of ways at least one woman and one man

$= {}_{45}P_4 - {}_{25}P_4 - {}_{20}P_4$

$= 3,575,880 - 303,600 - 116,280$

$= 3,156,000$

c) Two calculations

Male President, Female VP

$= 25 \times 20 \times 43 \times 42$

President, VP, Total-2, Total-3

$= 903,000$

Female President, Male VP

$= 20 \times 25 \times 43 \times 42$

$= 903,000$

Total ways

$= 903,000 + 903,000$

$= 1,806,000$

12. Four letters are randomly selected from the alphabet. What is the probability that they are A, B, C, and D, in that order,

a) if repetition is permitted?

b) if repetition is not permitted?

a) Can choose from all 26 letters each time

$P(ABCD) = P(A) \times P(B) \times P(C) \times P(D)$

$= \frac{1}{26} \times \frac{1}{26} \times \frac{1}{26} \times \frac{1}{26}$

$= \frac{1}{456,976}$

b) $P(ABCD) = P(A) \times P(B|A) \times P(C|AB) \times P(D|ABC)$

$= \frac{1}{26} \times \frac{1}{25} \times \frac{1}{24} \times \frac{1}{23}$

$= \frac{1}{358,800}$

13. Ten people each randomly select a number between 1 and 20. What is the probability that at least two of them select the same number?

$= 1 - P(\text{no two numbers the same})$

$= 1 - \left(\frac{20}{20} \times \frac{19}{20} \times \frac{18}{20} \times \frac{17}{20} \times \ldots \times \frac{12}{20} \times \frac{11}{20}\right)$

$= 1 - 0.0655$

$= 0.9345$

$\Rightarrow$ A 93.45% chance that at least two of the ten students will select the same number.

OR Dependent events....

Sample space is 20^{10}

Choosing different numbers in ${}_{20}P_{10}$ ways

$\Rightarrow P(\text{all different}) = \frac{{}_{20}P_{10}}{20^{10}}$

$P(\text{at least 2 the same}) = 1 - P(\text{all different})$

$= 1 - \frac{{}_{20}P_{10}}{20^{10}}$

$= 0.9345$