Permutations Review

Learning Goals

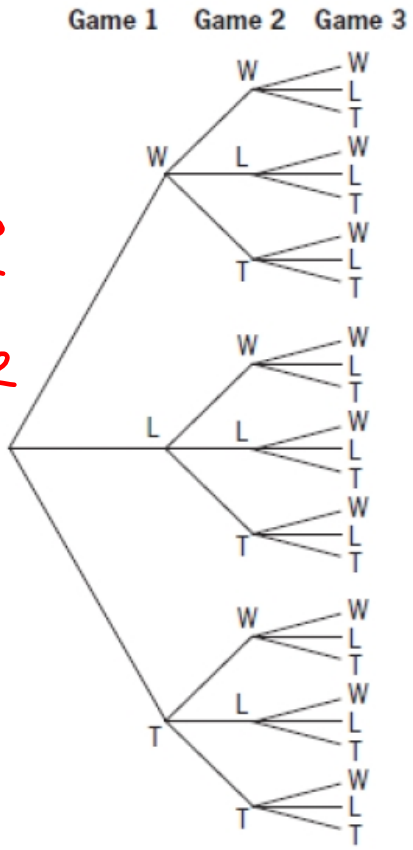
Section	After this section, I can
2.1	• make lists, charts, and tree diagrams to organize counting
2.2	• use the fundamental counting principle for counting and to solve problems
2.3	• see how using permutations has advantages over other counting techniques • solve simple problems using techniques for counting permutations • write permutation solutions using proper mathematical notation
2.4	• use the rule of sum to solve counting problems
2.5	• solve probability problems using counting principles for situations with equally likely outcomes

MHR Page 96 #s 1, 3 - 5, 7, 9, 10 & 12 - 14

Solutions

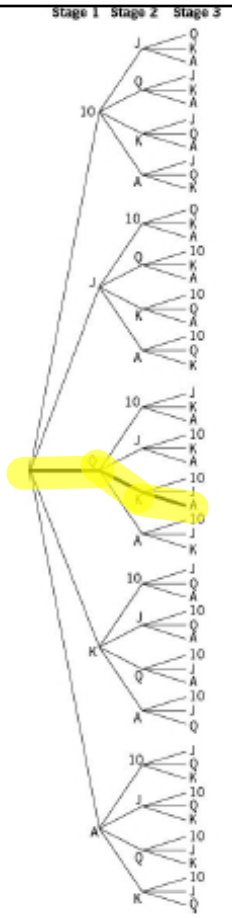
1. Draw a tree diagram showing all the possible outcomes (win, loss, or tie) in three games between two hockey teams. How many possible outcomes are there?

There are 3x3x3
= 27 outcomes
that are possible



3. The heart honour cards (10, J, Q, K, A) are removed from a standard deck. Three cards are randomly selected from the heart honour cards without replacement.
   a) Illustrate all the possible outcomes using a tree diagram.
   b) Highlight the path that indicates the run queen of hearts, king of hearts, ace of hearts).
   c) How many possible outcomes are there?

There 5x4x3
= 60 outcomes
that are possible



4. A home security code requires five digits to be entered on a keypad.

a) How many distinct security codes are possible?

b) Sarah reset her security code but has forgotten it. If it takes her eight seconds per attempt, what is the maximum time it would take for her to find the correct code?

10 possible digits to choose from
Repeats are permitted.

a) $= 10^5 = 100,000$ distinct codes

b) $8 \times 100,000 = 800,000$ seconds

$\Rightarrow \frac{800,000}{60} = 13,333\frac{1}{3}$ min

$\Rightarrow \frac{13,333\frac{1}{3}}{60} = 222\frac{2}{9}$ hours

$\Rightarrow \frac{222\frac{2}{9}}{24} = 9.26$ days

5. When ordering a gaming computer online, Ryan has three choices for processors, four choices for size of RAM, five choices for the video card, three choices for the hard drive, and two choices for the sound card.

a) How many choices does Ryan have when configuring his computer?

b) If there were an additional choice for the video card, how would it affect the total number of choices? Explain the difference.

a) Processor × RAM × Video cards × Hard Disks × Sound cards

$= 3 \times 4 \times 5 \times 3 \times 2$

$= 360$ choices

b) Extra video card

$\Rightarrow 3 \times 4 \times 6 \times 3 \times 2$

$= 432$ choices

7. How many ways are there for a company to assign three different jobs to three of its five employees?

There are $5P_3$ ways

$= \frac{5!}{(5-3)!}$

$= \frac{5!}{2!}$

$= 60$

Could also have
5 ways to assign #1
4 ways to assign #2
3 ways to assign #3

$\Rightarrow 5 \times 4 \times 3 = 60$ ways

9. A bookstore clerk is arranging seven novels, four plays, and five poetry books in a display case. Each type of book remains in its own group, but the groups can be in any order. In how many ways could she arrange the books?

3 "groups" need arranging.
Books within each group also need arranging.

$\Rightarrow (3!)(7!)(4!)(5!)$

Groups, Novels, Plays, Poetry

$= 87,091,200$ arrangements

10. In how many ways could the letters in the word STORAGE be arranged if the vowels must remain in
    a) even positions?
    b) odd positions?
    c) even or odd positions?

4 3 3 2 2 1 1

Vowels $= 3 \times 2 \times 1 = 3! = 6$

Consonants $= 4 \times 3 \times 2 \times 1 = 4! = 24$

Vowels AND consonants $= 6 \times 24$
$= 144$ ways

10. In how many ways could the letters in the word STORAGE be arranged if the vowels must remain in
    a) even positions?
    b) odd positions?
    c) even or odd positions?

7 letters, 3 are vowels
Even positions: 2, 4, 6
Odd positions: 1, 3, 5, 7

a) Arrange vowels in ${}_3P_3$ ways
Arrange consonants in ${}_4P_4$ ways
$\Rightarrow {}_3P_3 \times {}_4P_4 = 144$ ways

b) Arrange vowels in ${}_4P_3$ ways
Arrange consonants in ${}_4P_4$ ways
$\Rightarrow {}_4P_3 \times {}_4P_4 = 576$ ways

c) Even or odd $\Rightarrow$ Vowels can be anywhere
$\Rightarrow 7!$ arrangements
$= 5040$

12. How many five-digit even numbers can be formed using all the digits 0, 1, 2, 3, and 4?

Think in terms of where the zero will be and count to the left

Zero in ones position

$\Rightarrow 1 \times 2 \times 3 \times 4 \times 1 = 24$

Zero in tens position ← zero

$\Rightarrow 1 \times 2 \times 3 \times 1 \times 2$ ← digit must be 2 or 4

$= 12$

Zero in hundreds position

$\Rightarrow 1 \times 2 \times 1 \times 3 \times 2$ ← zero

$= 12$

Zero in thousands position

$\Rightarrow 1 \times 1 \times 2 \times 3 \times 2$

$= 12$

Zero in ten thousands position $\Rightarrow$ can't happen

Total ways

$= 24 + 12 + 12 + 12$

$= 60$ ways

13. For a gift exchange, 10 people's names are written on slips of paper and placed in a bowl. The slips of paper are mixed up, and each person selects one name.

a) What is the probability that everyone selects their own name?

b) What is the probability that nobody selects his or her own name?

a) $= \frac{1}{10} \times \frac{1}{9} \times \frac{1}{8} \times \frac{1}{7} \times \ldots$

$= \frac{1}{10!}$

$= \frac{1}{3{,}628{,}800}$

b) Use the complement? $\Rightarrow$ NO [includes one or two or.... picking their own names]

$= \frac{9}{10} \times \frac{8}{9} \times \frac{7}{8} \times \frac{6}{7} \times \ldots$

$= \frac{9!}{10!}$

$= \frac{1}{10}$

14. Six different coloured balls are placed in a box. Kendra and Abdul each select a ball without replacement.

a) What is the probability that Kendra selects the green ball and Abdul selects the red ball?

b) What is the probability that Kendra selects the green ball and Abdul does not select the red ball?

c) What is the probability that Kendra does not select the green ball and Abdul does not select the red ball?

$= \frac{1}{6} \times \frac{1}{5} = \frac{1}{30}$

$= \frac{1}{6} \times \frac{4}{5} = \frac{4}{30} = \frac{2}{15}$

Use the complement? Yes

$= 1 - \frac{1}{30}$

$= \frac{29}{30}$