Probability Problems Using Permutations

Lesson objectives

- I can solve probability problems using counting principles for situations with equally likley outcomes

1.1

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Warm Up

Event	
hoping for a king to be dealt, given that a king has already been dealt	dependent
Jonathan Toews has a 17.6% probability of scoring	statistical
two rolls of a single die	independent
a number is even, a number is odd	complements
dealing three cards from a standard deck	mutually exclusive
the results on a green die or a red die	rule of sum
the results on a green die and a red die	fundamental counting principle
a card is a heart, a card is a spade	mutually exclusive

Using the following words match them up to the events listed above: mutually exclusive (you can use this twice), rule of sum, statistical, independent, dependent, complements, fundamental counting principle.

In this last section we are starting to apply probability to more complex problems. Look to identify the sample space (all possible outcomes) and then the number of elements that you are looking for.

Example 1

Independent Trials

Software for generating multiple choice tests randomly assigns A, B, C, or D as the correct answer. On a 10-question test, what is the probability that all 10 questions have C as the correct answer?

$P(C) = \frac{1}{4}$

$P(10\ Cs) = \left(\frac{1}{4}\right)^{10} = \frac{1}{1{,}048{,}576}$

Your Turn

A street illusionist asks five people to each secretly write a number between 1 and 100 on a card. Incredibly, they all write the same number.

a) What is the probability of this occurring?

b) Relate your answer to part a) to the probability of rolling a six on a standard die five times in a row.

$P(\text{correct}) = \frac{1}{100}$

a) $P(\text{all 5 same}) = \left(\frac{1}{100}\right)^5 = \frac{1}{10{,}000{,}000{,}000}$

b) $P(6) = \frac{1}{6}$

$P(5\ \text{sixes}) = \left(\frac{1}{6}\right)^5 = \frac{1}{7776}$

Example 2

Dependent Trials

Eight people on a waiting list for advance tickets to a concert have been selected to choose their seats. What is the probability they will have been notified in order from youngest to oldest?

Arrange 8 people in 8! ways

Arrange youngest to oldest in 1 way

$P(\text{young to old}) = \frac{1}{8!} = \frac{1}{40320}$

Your Turn

Four students, one from each of grades 9, 10, 11, and 12, line up to pose for a photograph. What is the probability that they will be in order of their grades?

Arrange 4 students in 4! ways

Arrange in grade order 9 to 12 in 1 way

$P(\text{grade order}) = \frac{1}{4!} = \frac{1}{24}$

Example 3

Ordered Selections

Logan selects three cards in order, without replacement, from a standard deck. What is the probability that he selects a king, then two queens?

Arrangements of 3 cards $_{52}P_3$

Arrangements of 1 King $_4P_1$

Arrangements of 2 Queens $_4P_2$

$P(K,Q,Q) = \frac{_4P_1 \times {_4P_2}}{_{52}P_3} = \frac{4 \times 12}{132600} = \frac{2}{5525}$

Your Turn

Kylie selects five cards.

a) What is the probability that she selects three aces followed by two jacks?

b) What is the probability that Kylie selects two hearts followed by three clubs?

a) Arrangements of 5 cards $= {_{52}P_5}$

Arrangements of 3 Aces $= {_4P_3}$

Arrangements of 2 Jacks $= {_4P_2}$

$P(AAAJJ) = \frac{_4P_3 \times {_4P_2}}{_{52}P_5} = \frac{24 \times 12}{311875200} = \frac{1}{1{,}082{,}900}$

b) Arrangements of 2 Hearts $= {_{13}P_2}$

Arrangements of 3 Clubs $= {_{13}P_3}$

$P(HHCCC) = \frac{_{13}P_2 \times {_{13}P_3}}{_{52}P_5} = \frac{156 \times 1716}{311875200}$

$= \frac{143}{166600}$

Example 4

The Birthday Problem

There are 30 students in Wayne's class.

a) What is the probability that no two people have the same birthday?

b) What is the probability that at least two students share the same birthday?

a) Choose 30 different days from 365 $= {_{365}P_{30}}$

# of arrangements $= 365^{30}$

$P(\text{all different}) = \frac{_{365}P_{30}}{365^{30}} \simeq 0.2937$

b) $P(\text{at least two the same}) = 1 - (\text{none the same})$

$= 1 - 0.2937$

$= 0.7063$

Your Turn

From a group of 16 people, what is the probability that

a) none share a birthday?

b) at least two of them share the same birthday?

365^{16} arrangements

a) $_{365}P_{16}$ arrangements for none sharing

$\Rightarrow P(\text{none sharing}) = \frac{_{365}P_{16}}{365^{16}} \simeq 0.7164$

b) $P(\text{at least two sharing}) = 1 - P(\text{none sharing})$

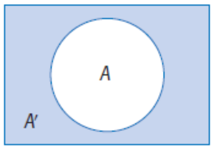
$= 1 - 0.7164$

$= 0.2836$

Key Concepts

- You can calculate the probability of an event using $P = \frac{n(A)}{n(S)}$, where $n(A)$ is the number of successful outcomes and $n(S)$ is the number of outcomes in the sample space.
- If the trials are dependent, you can use permutations in the calculations.
- To use the indirect method, subtract the probability of the complement from 1.

  $P(A) = 1 - P(A')$



$P(A) = 1 - P(A')$

Homework

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Use ${}_nP_r$ when events are DEPENDENT, as in can't be chosen again.

Use m^n when events are INDEPENDENT, as in can be chosen again, where m = # of options and n = # of trials.