Solutions

1. Determine the total number of arrangements of three or four toys from a basket of eight different toys.

$${}_8P_3 + {}_8P_4 = 336 + 1680$$
$$= 2016$$

choose 3 from 8

choose 4 from 8

2. a) How many ways are there to roll a sum of 7 or 11 on two dice?

b) How many ways are there to roll doubles or a sum divisible by three on two dice?

a) Sum of 7 = 6 ways ((1,6)(2,5)(3,4) (4,3)(5,2)(6,1))

Sum of 11 = 2 ways ((5,6)(6,5))

Sum 7 or 11 = 6 + 2
= 8 ways

b) doubles = 6 ways

divisible by 3 ⇒ totals of 3, 6, 9, 12

sum of 3 = 2 ways
sum of 6 = 5 ways
sum of 9 = 4 ways
sum of 12 = 1 way
} 12 ways

But (3,3) and (6,6) are in both sets of outcomes (non-mutually exclusive) so they need to be subtracted from the total.

⇒ n(doubles) + n(divisible by 3) − n(doubles divisible by 3)
= 6 + 12 − 2 = 16 ways

3. A game has players roll either one or two standard dice. Which is the total number of possible different outcomes?

A 42
B 36
C 18
D 12

Roll one die = 6 outcomes

Roll two dice = 6 × 6 = 36 outcomes

⇒ Rolling one OR two = 6 + 36
= 42 different outcomes

4. Which is the total number of arrangements of the digits 1, 2, 3, 4, 5, if the even digits must not be together?

A 120
B 24
C 48
D 72

Total arrangements $= 5!$

Even together $= 2!4!$

2,4 and 1,3,5 and "2,4"

$\Rightarrow$ Not together $= 5! - (2!4!)$

$= 120 - 48$

$= 72$

6. **Application** A motorcycle licence plate consists of two or three letters followed by four digits. How many licence plates can be made?

26 letters available (A → Z)

10 digits avaible (0 → 9)

2 letters with 4 digits

$= 26 \times 26 \times 10 \times 10 \times 10 \times 10$

$= 26^2 \times 10^4$

$= 6760000$

3 letters with 4 digits

$= 26^3 \times 10^4$

$= 175,760,000$

Total $= 6760000 + 175760000$

$= 182,520,000$

7. A security code consists of either five or six different letters. How many distinct security codes are possible?

5 different letters $\Rightarrow$ no repeats

$= 26 \times 25 \times 24 \times 23 \times 22$ $\left[\frac{26!}{21!}\right]$

$= 7,893,600$

6 different letters $\Rightarrow$ no repeats

$= 26 \times 25 \times 24 \times 23 \times 22 \times 21$ $\left[\frac{26!}{20!}\right]$

$= 165,765,600$

Total $= 7,893,600 + 165,765,600$

$= 173,659,200$