Rule of Sum

Lesson objectives

- I can use the rule of sum to solve counting problems

1.1

Lesson objectives | Teachers' notes | Lesson notes

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Warm Up

Many grade 12 students consider attending university. Sidney is applying to Waterloo for math or engineering, Queen's for physics, chemistry, or engineering, and Laurentian for science or engineering. How many program choices does Sidney have in total?

WM, WE,
QP, QC, QE,
LS, LE
⇒ 7

Jenna is applying to Windsor for business or economics, York for commerce or business, and Ottawa for economics, political science, or business. How many program choices does Jenna have in total?

WB, WE
YC, YB
OE, OPS, OB
⇒ 7

Complete the "Investigate" on Page 82

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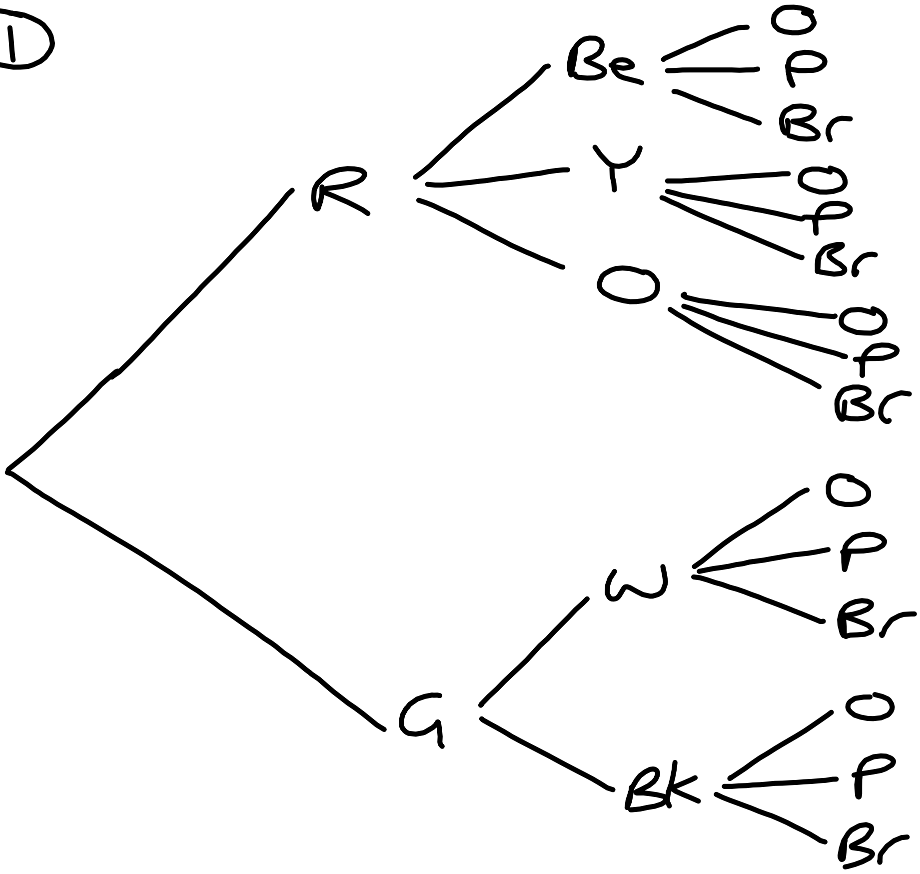

2 Red first → 9 outcomes

3 Green first → 6 outcomes

4 15 outcomes in total

5 Add the answers to 2 and 3 to give the answer to 4

6 $n(A \text{ or } B) = n(A) + n(B)$

7 a) $n(\text{coin OR die}) = n(\text{coin}) + n(\text{die})$

$= 2 + 6$

$= 8$ outcomes

b) $n(\text{coin AND die}) = n(\text{coin}) \times n(\text{die})$

$= 2 \times 6$

$= 12$ outcomes

Example 1

Use the Rule of Sum

Note: OR ⇒ ADD

At an international conference, either eight or nine countries may attend. In how many different arrangements could the countries' flags be flown?

If 8 countries $= 8!$

If 9 countries $= 9!$

Total $= 8! + 9!$

$= 40320 + 362880$

$= 403200$

Your Turn

At the conference in Example 1,

a) in how many different arrangements could the flags be flown if seven, eight, or nine countries attend?

$= 7! + 8! + 9! = 408240$

b) in how many different arrangements could the flags be flown if the host country's flag is always on the far left?

If on the left it has only one position, so we perm the others ⇒ $1!6! + 1!7! + 1!8!$

$= 46080$

$${}_nP_r = \frac{n!}{(n-r)!}$$

n = # of total choices

r = # of favourable choices

$(n-r)$ = # of unfavourable choices

Example 2

Use the Principle of Inclusion and Exclusion

Three players are playing the card game Pass the Ace. Each player receives one card. In how many ways could the cards all be face cards or red cards?

All face cards $\Rightarrow {}_{12}P_3 = \frac{12!}{(12-3)!} = 1320$

All red cards $\Rightarrow {}_{26}P_3 = \frac{26!}{(26-3)!} = 15600$

Red and face $\Rightarrow {}_6P_3 = \frac{6!}{(6-3)!} = 120$

$\Rightarrow 1320 + 15600 - 120$

$= 16800$

overlap

Your Turn

Three players each cut one card from a standard deck. If order is important, in how many ways could they be

a) all hearts?

b) all aces?

c) all aces or hearts?

a) ${}_{13}P_3 = \frac{13!}{(13-3)!} = 1716$

b) ${}_4P_3 = \frac{4!}{(4-3)!} = 24$

c) ${}_4P_3 + {}_{13}P_3 - {}_1P_1$

Ace of hearts

$= 1716 + 24 - 1$

$= 1739$

Example 3

Use the Indirect Method

The 12 members of a basketball team are lining up for their medals after a tournament. In how many ways can this be done

a) if there is no restriction?

b) if the captain and assistant captain must be together?

c) if the captain and assistant captain must not be together?

indirect method

- subtract the number of unwanted outcomes from the total number of outcomes without restrictions

a) $12! = 479{,}001{,}600$

b) Treat C and AC as one player

$= 2!\,11! = 79{,}833{,}600$

c) All ways except together

$= 12! - (2!\,11!) = 399{,}168{,}000$

Your Turn

In how many ways could the letters in the word FACTOR be arranged so that the vowels are not together?

Total arrangements $= 6!$ (6 letters)

Ways with vowels together $= 2!\,5!$

2 vowels "AO" treated as one letter

arranging 5 objects F, C, T, R and "AO"

$\Rightarrow$ Ways with vowels NOT together $= 6! - (2!\,5!)$

$= 720 - 240$

$= 480$

Key Concepts

- The rule of sum states that if one mutually exclusive event can occur in m ways, and a second can occur in n ways, then one **or** the other can occur in $m + n$ ways.
- If two events are not mutually exclusive, the principle of inclusion and exclusion needs to be considered:

  $n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$.
- To reduce calculations, consider using the indirect method, which involves subtracting the unwanted event from the total number of outcomes in the sample space: $n(A) = n(S) - n(A')$.

Homework

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