Solutions

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1. Evaluate.
   a) $9!$
   b) $\dfrac{12!}{5!}$
   c) $_7P_7$
   d) $_8P_5$

a) $9! = 362,880$

b) $\dfrac{12!}{5!} = 3,991,680$

c) $_7P_7 = \dfrac{7!}{(7-7)!} = \dfrac{7!}{0!} = 7! = 5040$

d) $_8P_5 = \dfrac{8!}{(8-5)!} = \dfrac{8!}{3!} = 6720$

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2. Write in factorial form.

a) $_6P_4$
b) $_{15}P_6$
c) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
d) $8 \times 7 \times 6 \times 5$
e) $n(n-1)(n-2)(n-3)$
f) $(n+1) \times (n) \times (n-1) \times \ldots \times 3 \times 2 \times 1$

$$_nP_r = \frac{n!}{(n-r)!}$$

a) $6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!}$

b) $15P_6 = \frac{15!}{(15-6)!} = \frac{15!}{9!}$

c) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 7!$

d) $\frac{8 \times 7 \times 6 \times 5 \times \cancel{4 \times 3 \times 2 \times 1}}{\cancel{4 \times 3 \times 2 \times 1}} = \frac{8!}{4!}$

e) $\frac{n(n-1)(n-2)(n-3)\cancel{(n-4)\ldots(2)(1)}}{\cancel{(n-4)\ldots(2)(1)}} = \frac{n!}{(n-4)!}$

f) $(n+1)(n)(n-1)\ldots(2)(1) = (n+1)!$

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3. Express in the form $_nP_r$.

a) $6!$
b) $91 \times 90 \times 89 \times 88 \times 87 \times 86$
c) $\frac{18!}{12!}$

$$_nP_r = \frac{n!}{(n-r)!}$$

a) $6! = \frac{6!}{0!} = \frac{6!}{(6-6)!} = 6P_6$

b) $\frac{91 \times 90 \times 89 \times 88 \times 87 \times 86 \times \cancel{85 \times \ldots (2)(1)}}{\cancel{85 \times \ldots (2)(1)}}$

$= \frac{91!}{85!} = \frac{91!}{(91-6)!} = 91P_6$

c) $\frac{18!}{12!} = \frac{18!}{(18-6)!} = 18P_6$

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4. Which is the correct simplification of $\frac{96!}{24!}$?

   A   4!

   B   4

   C $_{96}P_{72}$

   D $_{96}P_{24}$

$$_nP_r = \frac{n!}{(n-r)!}$$

$$\frac{96!}{24!} = \frac{96!}{(96-72)!} = {}_{96}P_{72}$$

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5. Which is the correct number of permutations of five items from a list of nine items?

   A   126

   B   15 120

   C   45

   D   59 049

$_nP_r$

Total choices

choices you want

$$= {}_9P_5$$
$$= \frac{9!}{(9-5)!} = \frac{9!}{4!} = 15120$$

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6. There are 15 teams competing in a synchronized swimming competition. In how many ways could first, second, and third place be awarded?

$$15 P_3 = 2730 \text{ ways}$$

# of teams   # of medallists

OR

15 could place 1ST
14 could place 2ND
13 could place 3RD

$\Rightarrow 15 \times 14 \times 13 = 2730 \text{ ways}$

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7. A club has 18 members. In how many ways could a president, vice president, treasurer, and secretary be elected?
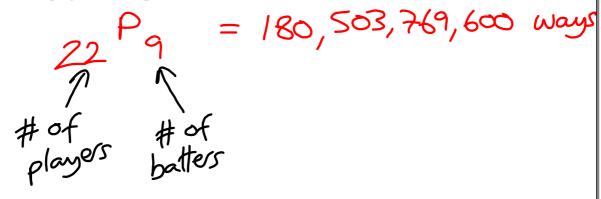
$$18 P_4 = 73,440 \text{ ways}$$

# of members   # of positions

18 could be President
17 could be VP
16 could be Treasurer
15 could be Secretary

$\Rightarrow 18 \times 17 \times 16 \times 15 = 73,440 \text{ ways}$

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8. There are 22 players on a baseball team. In how many ways could the batting order of nine players be assigned?

$$_{22}P_9 = 180,503,769,600 \text{ ways}$$

22 ← # of players

9 ← # of batters

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9. Write in simplest factorial form.
   a) $10 \times 9 \times 8 \times 7!$
   b) $99 \times 98 \times 97!$
   c) $90 \times 8!$
   d) $n(n-1)!$
   e) $(n+2)(n+1)n!$

a) $= 10! \quad (10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)$

b) $= 99! \quad (99 \times 98 \times 97 \times 96 \times 95 \times \ldots \times 2 \times 1)$

c) $= 10 \times 9 \times 8!$
   $= 10!$

d) $= n! \quad \left[n(n-1)(n-2)(n-3)\ldots(2)(1)\right]$

e) $= (n+2)! \quad \left[(n+2)(n+1)(n)(n-1)\ldots(2)(1)\right]$

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10. **Application** A salesperson needs to visit 15 different offices during the week.

   a) In how many ways could this be done?

   b) In how many ways could she visit four different offices on Monday?

   c) In how many ways could she visit three different offices each day from Monday to Friday?

a) $15! = 130,767,436,800$

b) $15 P_4 = \dfrac{15!}{(15-4)!} = \dfrac{15!}{11!} = 32,760$

c) $15 P_{15} = \dfrac{15!}{(15-15)!} = \dfrac{15!}{0!} = 130,767,436,800$

\# of offices

\# in one day 3×5 \# of days

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