Quadratic Functions Review

1. Properties of Quadratics
2. Converting Between Forms (Expanding and Factoring)
3. Finding the Zeros
4. Determining the Number of Zeros (Discriminant, Signs of a and k)
5. Max/Min Values
6. Word Problems
7. Write the Equation
8. Solving Linear-Quadratic Systems
9. The 4 Transformations

Nelson Page 202 #s 1ac, 2abd, 3 - 5, 12 - 15 & 19 - 22

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Solutions

Nov 20-18:35

1. Consider the quadratic function
$f(x) = -3(x-2)^2 + 5.$

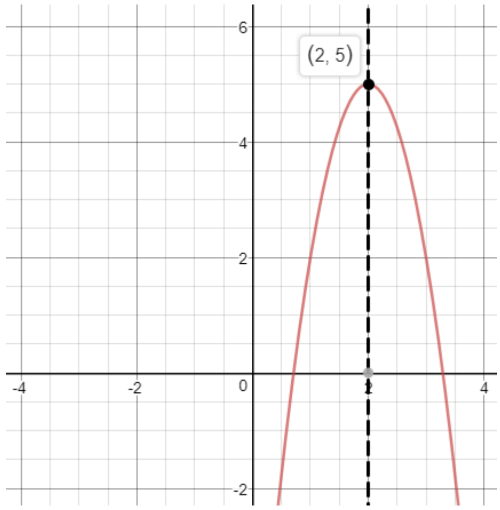
a) State the direction of opening, the vertex, and the axis of symmetry.

Opens down

Vertex is $(2,5)$

Axis of symmetry is $x = 2$

c) Graph the function.



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2. Consider the quadratic function
$f(x) = 4(x-2)(x+6).$

a) State the direction of opening and the zeros of the function.

Opens up

Zeros at $(2,0)$ and $(-6,0)$

b) Determine the coordinates of the vertex.

$h = \frac{r+s}{2}$

$h = \frac{2+(-6)}{2}$

$h = \frac{-4}{2} = -2$

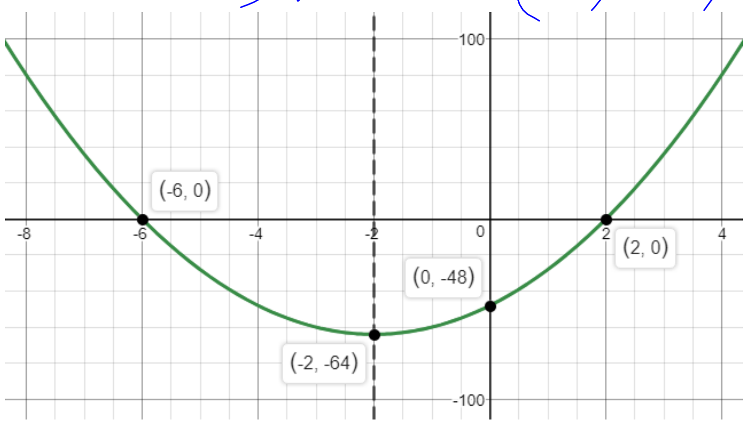
$k = 4(-2-2)(-2+6)$

$k = 4(-4)(4)$

$k = -64$

$\Rightarrow$ vertex is $(-2,-64)$

d) Graph the function.



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3. Determine the equation of the axis of symmetry of the parabola with points $(-5, 3)$ and $(3, 3)$ equally distant from the vertex on either side of it.

h - value is halfway between -5 and 3

$\Rightarrow h = \frac{-5+3}{2}$

$h = \frac{-2}{2}$

$h = -1$

$\Rightarrow$ Axis of symmetry is $x = 1$

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4. For each quadratic function, state the maximum or minimum value and where it will occur.

a) $f(x) = -3(x - 4)^2 + 7$

Max value of 7

Occurs at (4, 7)

b) $f(x) = 4x(x + 6)$

$h = \frac{r+s}{2} = \frac{0+-6}{2}$

$= \frac{-6}{2}$

$= -3$

$k = 4(-3)(-3+6)$

$k = 4(-3)(3)$

$k = -36$

$\Rightarrow$ min value of -36 occurs at (-3, -36)

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5. The height, $h(t)$, in metres, of the trajectory of a football is given by $h(t) = 2 + 28t - 4.9t^2$, where t is the time in flight, in seconds. Determine the maximum height of the football and the time when that height is reached.

Find time of vertex $\left(-\frac{b}{2a}\right)$

$$= \frac{-28}{2(-4.9)} = \frac{-28}{-9.8} = 2.86$$

$$h = 2 + 28(2.86) - 4.9(2.86)^2$$

$$h = 42$$

$\Rightarrow$ Max height of 42m is reached after 2.86 seconds

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12. Determine the x-intercepts of the quadratic function $f(x) = 2x^2 + x - 15$.

$ac = 2(-15) = -30$

1×-30
2×-15
3×-10
5×-6
6×-5
10×-3
15×-2
30×-1

$6 + -5 = 1$
$6 \times -5 = -30$

$$\Rightarrow 0 = 2x^2 + 6x - 5x - 15$$
$$0 = 2x(x+3) - 5(x+3)$$
$$0 = (x+3)(2x-5)$$

$x + 3 = 0$
$x = -3$

$2x - 5 = 0$
$\frac{2x}{2} = \frac{5}{2}$
$x = 2\frac{1}{2}$

$\Rightarrow$ x-intercepts are $(-3, 0)$ and $(2\frac{1}{2}, 0)$

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13. The population of a Canadian city is modelled by $P(t) = 12t^2 + 800t + 40\,000$, where t is the time in years. When $t = 0$, the year is 2007.

a) According to the model, what will the population be in 2020?

b) In what year is the population predicted to be 300 000?

a) $t = 0$ in 2007
So $t = 13$ in 2020 $[2020 - 2007 = 13]$

$P = 12(13)^2 + 800(13) + 40000$

$P = 52428$

b) $300000 = 12t^2 + 800t + 40000$

$0 = 12t^2 + 800t - 260000$

$(\div 4)$ $0 = 3t^2 + 200t - 65000$

$$t = \frac{-200 \pm \sqrt{200^2 - 4(3)(-65000)}}{2(3)}$$

$$t = \frac{-200 + \sqrt{820000}}{6} \qquad t = \frac{-200 - \sqrt{820000}}{6}$$

$t = 117.6$ $\qquad$ $t = -184.3$ Extraneous

$\Rightarrow$ Year $= 2007 + 117.6$
$= 2124.6$
$= 2125$

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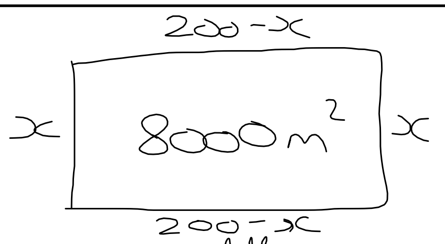
14. A rectangular field with an area of 8000 m² is enclosed by 400 m of fencing. Determine the dimensions of the field to the nearest tenth of a metre.



Let width $= x$
Perimeter $= 400$ m
$\Rightarrow$ length $= 200 - x$

Area = length × width

$8000 = (200 - x)(x)$

$8000 = 200x - x^2$

$x^2 - 200x + 8000 = 0$

$$x = \frac{200 \pm \sqrt{(-200)^2 - 4(1)(8000)}}{2(1)}$$

$$x = \frac{200 + \sqrt{8000}}{2} \qquad x = \frac{200 - \sqrt{8000}}{2}$$

$x = 144.7$ m $\qquad$ $x = 55.3$ m

$\Rightarrow$ Dimensions are 144.7m × 55.3m

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15. The height, $h(t)$, of a projectile, in metres, can be modelled by the equation $h(t) = 14t - 5t^2$, where t is the time in seconds after the projectile is released. Can the projectile ever reach a height of 9 m? Explain.

Find time of the vertex $\left(-\frac{b}{2a}\right)$

$$-\frac{b}{2a} = \frac{-14}{2(-5)} = \frac{-14}{-10} = 1.4 \text{ seconds}$$

$$\text{Max height} = 14(1.4) - 5(1.4)^2$$
$$= 9.8 \text{ metres}$$

$\Longrightarrow$ Yes, it can reach a height of 9m because the maximum height is 9.8m

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19. Describe the characteristics that the members of the family of parabolas $f(x) = a(x + 3)^2 - 4$ have in common. Which member passes through the point $(-2, 6)$?

Members of the VERTEX family. They all have the same vertex $(-3,-4)$

Using $(-2,6)$, solve for a

$$6 = a(-2+3)^2 - 4$$
$$6 = a(1)^2 - 4$$
$$10 = a$$

$\Longrightarrow y = 10(x+3)^2 - 4$ passes through $(-2,6)$

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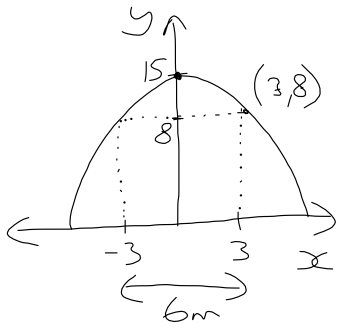
20. An engineer is designing a parabolic arch. The arch must be 15 m high, and 6 m wide at a height of 8 m.

a) Determine a quadratic function that satisfies these conditions.

b) What is the width of the arch at its base?



a) Vertex (h,k) is $(0,15)$

Point (x,y) is $(3,8)$

Write in vertex form

$8 = a(3-0)^2 + 15$

$8 = a(9) + 15$

$\frac{-7}{9} = \frac{9a}{9}$

$\Rightarrow y = -\frac{7}{9}(x)^2 + 15$

b) $0 = -\frac{7}{9}x^2 + 15$

$-15 = -\frac{7}{9}x^2$

$\frac{-135}{-7} = \frac{-7x^2}{-7}$

$19\frac{2}{7} = x^2$

$\pm\sqrt{19\frac{2}{7}} = x$

$x = 4.39$ and -4.39

$\Rightarrow$ width of arch $= 4.39 - (-4.39)$

$= 8.78\text{m}$

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21. Calculate the point(s) of intersection of $f(x) = 2x^2 + 4x - 11$ and $g(x) = -3x + 4$.

$2x^2 + 4x - 11 = -3x + 4$

$2x^2 + 4x + 3x - 11 - 4 = 0$

$2x^2 + 7x - 15 = 0$

$ac = 2(-15) = -30$

1 x -30
2 x -15
3 x -10
5 x -6
6 x -5
10 x -3
15 x -2
30 x -1

$10 + -3 = 7$

$10 \times -3 = -30$

$2x^2 + 10x - 3x - 15 = 0$

$2x(x+5) - 3(x+5) = 0$

$(x+5)(2x-3) = 0$

$x + 5 = 0$

$x = -5$

$2x - 3 = 0$

$\frac{2x}{2} = \frac{3}{2}$

$y = -3(-5) + 4$

$y = 19$

$y = -3\left(\frac{3}{2}\right) + 4$

$y = -\frac{1}{2}$

$\Rightarrow$ Points of intersection are $(-5, 19)$ and $(3/2, -1/2)$

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22. The height, $h(t)$, of a baseball, in metres, at time t seconds after it is tossed out of a window is modelled by the function $h(t) = -5t^2 + 20t + 15$. A boy shoots at the baseball with a paintball gun. The trajectory of the paintball is given by the function $g(t) = 3t + 3$. Will the paintball hit the baseball? If so, when? At what height will the baseball be?

$-5t^2 + 20t + 15 = 3t + 3$

$-5t^2 + 20t - 3t + 15 - 3 = 0$

$-5t^2 + 17t + 12 = 0$

$ac = -5(12) = -60$

$20 \times -3 = -60$

$20 + -3 = 17$

$-5t^2 + 20t - 3t + 12 = 0$

$-5t(t-4) - 3(t-4) = 0$

$(t-4)(-5t-3) = 0$

$t - 4 = 0$

$t = 4$

$-5t - 3 = 0$

$\frac{-5t}{-5} = \frac{3}{-5}$

$t = -3/5$

Extraneous

height $= 3(4) + 3$

$= 15m$

Yes, it will hit after 4 seconds at a height of 15m.

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