Quadratic Functions Review

1. Properties of Quadratics
2. Converting Between Forms (Expanding and Factoring)
3. Finding the Zeros
4. Determining the Number of Zeros (Discriminant, Signs of a and k)
5. Max/Min Values
6. Word Problems
7. Write the Equation
8. Solving Linear-Quadratic Systems
9. The 4 Transformations

Nelson Page 202 #s 1ac, 2abd, 3 - 5, 12 - 15 & 19 - 22

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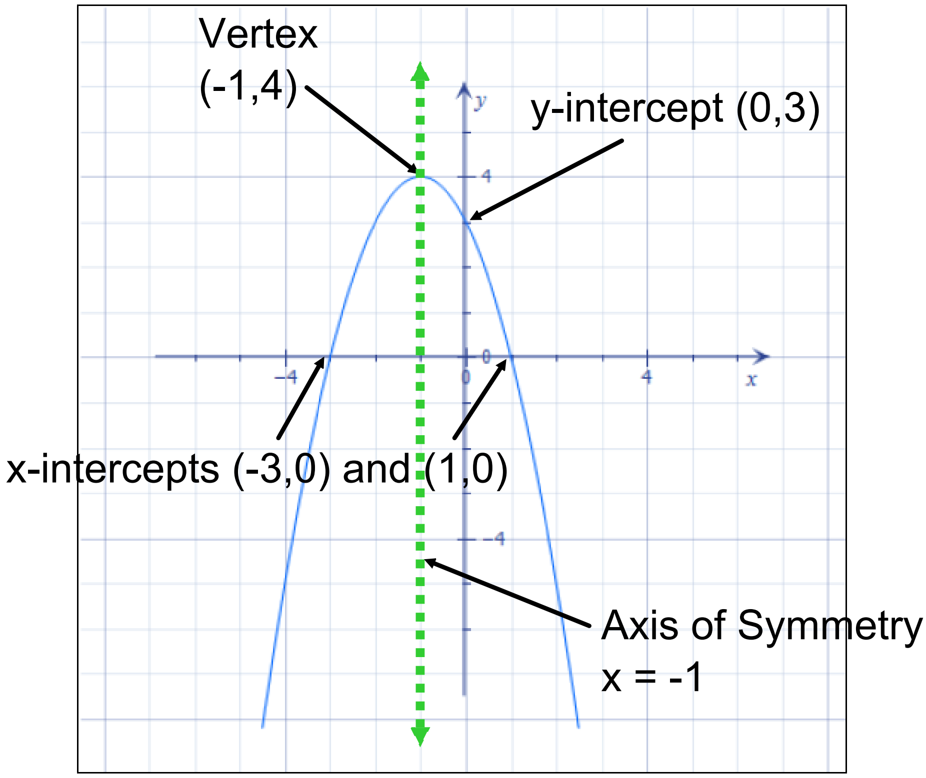
1. Important points on a parabola:

$y = -(x+1)^2 + 4$ or $y = -(x+3)(x-1)$ or $y = -x^2 - 2x + 3$



Vertex (-1,4)

y-intercept (0,3)

x-intercepts (-3,0) and (1,0)

Axis of Symmetry x = -1

2. Standard Form

$$y = ax^2 + bx + c$$

Sign of "a" gives the direction of opening
- positive, opens up (smiley face)
- negative, opens down (sad face)

"c" is the value of the y-intercept

3. Factored Form

$$y = a(x - r)(x - s)$$

Sign of "a" gives the direction of opening
- positive, opens up (smiley face)
- negative, opens down (sad face)

"r" and "s" give the values of the x-intercepts. They have the opposite sign of how they appear in the brackets.

- Writing the equation:

Use the x-intercepts for "r" and "s" and another point on the parabola for x and y. Substitute into the general factored form, solve for "a", and then write the equation.

4. Vertex Form

$$y = a(x - h)^2 + k$$

Sign of "a" gives the direction of opening
- positive, opens up (smiley face)
- negative, opens down (sad face)

"h" and "k" give, respectively, the x and y values of the vertex. Again values going in and out of the brackets will change sign.

- Writing the equation:

Use the vertex for "h" and "k" and another point on the parabola for x and y. Substitute into the general vertex form, solve for "a", and then write the equation.

5. Word Problems

x-intercepts	y-intercept	vertex
Break-even	Initial height	Maximum value Minimum value
Zero profit		
Ball hitting the ground		

Converting from Factored to Standard Form

Expand the brackets by either using the chart or using FOIL.

F: First times First

O: Outside times Outside

I: Inside times Inside

L: Last times Last

$(3a + 2b)(2a - 5b)$

$= 6a^2 - 15ab + 4ab - 10b^2$

$= 6a^2 - 11ab - 10b^2$

	$3a$	$2b$
$2a$	$6a^2$	$4ab$
$-5b$	$-15ab$	$-10b^2$

Converting from Vertex to Standard Form

Expand the double bracket (multiply it by itself) and then simplify.

$$3(x - 5)^2 + 8$$

$3(x - 5)(x - 5) + 8$

$= 3(x^2 - 5x - 5x + 25) + 8$

$= 3(x^2 - 10x + 25) + 8$

$= 3x^2 - 30x + 75 + 8$

$= 3x^2 - 30x + 83$

	x	-5
x	x^2	$-5x$
-5	$-5x$	25

$= 3(x^2 - 10x + 25) + 8$

Converting from Standard to Factored Form

This is known as factoring.

1. Common factor (if possible).
2. Find (a)(c).
3. Find two numbers that multiply to make (a)(c) that also add up to make b.
4. Fill in the four spots in the chart or decompose the b term so that it can be factored.
5. Chart: factor horizontally and vertically. Decomposition: factor by grouping. Note: if a negative term occurs first, take a negative factor.
6. Write your factored answer remembering to put the common factor in front of the brackets if you removed a common factor in step one.

$x^2 + 8x + 12$

(a)(c) = 12 Factors
b = 8 1 x 12
[✓ 2 x 6]
3 x 4

$= x^2 + 2x + 6x + 12$
$= x(x + 2) + 6(x + 2)$
$= (x + 2)(x + 6)$

$6x^2 - 21x + 15$

Common factor: 3

$= 3(2x^2 - 7x + 5)$

(a)(c) = 10 Factors
b = -7 -1 x -10
[✓ -2 x -5]

	x	-1
$2x$	$2x^2$	$-2x$
-5	$-5x$	5

$= 3(2x - 5)(x - 1)$

Converting from Standard to Vertex Form

This process is known as completing the square.

1. Factor out a from the x^2 and x terms.
2. Find half of the new b value and "square it, add it" and "square it, subtract it".
3. Now remove the "subtract it" term from the bracket, remembering to multiply it by the a value.
4. Factor the bracket.
5. Simplify the constant term.

$y = x^2 + 6x + 12$ $\quad\quad$ $y = -2x^2 + 16x - 15$

Complete the square

Factor a from the x^2 and x terms

$y = 1(x^2 + 6x) + 12$ $\quad\quad$ $y = -2(x^2 - 8x) - 15$

Find half of the b value, square it, add it, square it, subtract it

$y = 1(x^2 + 6x + (3)^2 - (3)^2) + 12$ $\quad\quad$ $y = -2(x^2 - 8x + (-4)^2 - (-4)^2) - 15$

Remove the subtracted term from the bracket: remember to multiply it by a

$y = 1(x^2 + 6x + (3)^2) - (1)(3)^2 + 12$

$y = -2(x^2 - 8x + (-4)^2) - (-2)(-4)^2 - 15$

Factor to make a perfect square

$y = 1(x + 3)^2 - (1)(3)^2 + 12$ $\quad\quad$ $y = -2(x - 4)^2 - (-2)(-4)^2 - 15$

Simplify the constant terms

$y = 1(x + 3)^2 - 9 + 12$ $\quad\quad$ $y = -2(x - 4)^2 - (-32) - 15$

$y = (x + 3)^2 + 3$ $\quad\quad$ $y = -2(x - 4)^2 + 17$

Converting from Factored to Vertex Form

We could expand the double bracket to convert to standard form and then complete the square.

Alternatively, we can use h = (r+s)/2 to find h and then sub this into the expression for x to find k. As the a value does not change, we can now write in vertex form.

$2(x - 3)(x + 5)$

Expand and simplify

$= 2(x^2 + 5x - 3x - 15)$

$= 2(x^2 + 2x - 15)$

$= 2x^2 + 4x - 30$

Now complete the square

$= 2(x^2 + 2x) - 30$

$= 2(x^2 + 2x + (1)^2 - (1)^2) - 30$

$= 2(x^2 + 2x + (1)^2) - (2)(1)^2 - 30$

$= 2(x + 1)^2 - 2 - 30$

$= 2(x + 1)^2 - 32$

$2(x - 3)(x + 5)$

Brackets = 0, solve for x

$x - 3 = 0$ $\quad$ $x + 5 = 0$

$x = 3$ $\quad$ $x = -5$

Vertex lies halfway between the x-intercepts

$h = \frac{r + s}{2}$

$h = \frac{3 + (-5)}{2}$

$h = -1$

Sub into equation for x

$2(x - 3)(x + 5)$

$= 2((-1) - 3)((-1) + 5)$

$= 2(-4)(4)$

$= -32$

Vertex form: $a(x - h)^2 + k$

$= 2(x + 1)^2 - 32$

Converting from Vertex to Factored Form

For this we would need to expand our repeated bracket and then simplify.

Then we can look to factor this expression using the usual (a)(c) method.

$$(x+1)^2 - 9$$

$$= (x+1)(x+1) - 9$$
$$= x^2 + x + x + 1 - 9$$
$$= x^2 + 2x - 8$$

$$x^2 + 2x - 8$$

(a)(c) = -8

b = 2

Factors

-1 x 8

$\boxed{-2 \times 4}$

$$= x^2 - 2x + 4x - 8$$
$$= x(x-2) + 4(x-2)$$
$$= (x-2)(x+4)$$

Solutions of a Quadratic

We can do this in three ways:

1. Factor - once factored, set each bracket to zero and solve
2. Vertex form - use SAMDEB to solve the equation
3. Quadratic formula - if in standard form and it won't factor

If asked for the **NUMBER** of solutions...

1. If in vertex form, draw a sketch using your knowledge of the direction of opening and where the vertex is OR analyze the signs of a and k (same = 0, different = 2, k = 0 has 1)
2. If in standard form, calculate the value of the discriminant

If $b^2 - 4ac > 0$ then there are 2 solutions

If $b^2 - 4ac = 0$ then there is 1 solution

If $b^2 - 4ac < 0$ then there are 0 solutions

Solutions of a Quadratic

Standard

Factor the equation (Don't forget to common factor!)

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $ax^2 + bx + c = 0$

Factored

Set each factor equal to zero and solve for x

Vertex

Convert to standard form and then factor

OR

Isolate for x using SAMDEB

Examples - Find the roots of the following:

$y = 3(x - 2)^2 - 18$

$0 = 3(x - 2)(x - 2) - 18$

$0 = 3(x^2 - 2x - 2x + 4) - 18$

$0 = 3(x^2 - 4x + 4) - 18$

$0 = 3x^2 - 12x + 12 - 18$

$0 = 3x^2 - 12x - 6$

$0 = x^2 - 4x - 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - (-8)}}{2}$$

$$x = \frac{4 \pm \sqrt{24}}{2}$$

x = 4.449 and x = -0.449

$y = 3(x - 2)^2 - 18$

$0 = 3(x - 2)^2 - 18$

$$\frac{18}{3} = \frac{3(x - 2)^2}{3}$$

$6 = (x - 2)^2$

$\pm\sqrt{6} = x - 2$

$2 \pm\sqrt{6} = x$

x = 4.449 and x = -0.449

Examples

For what value(s) of "a" can $y = ax^2 - 3x + 1$ have 2 solutions?

For there to be two solutions the discriminant, $b^2 - 4ac$, needs to be greater than zero.

$b^2 - 4ac > 0$

$(-3)^2 - 4(a)(1) > 0$

$9 - 4a > 0$

$\frac{9}{4} > \frac{4a}{4}$

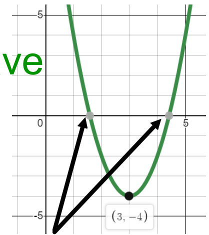
$2.25 > a$

It will have 2 solutions when a is less than 2.25

How many solutions are there for:

$y = 2(x - 3)^2 - 4$

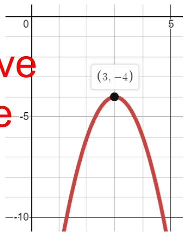
a & k have different signs



2 solutions

$y = -2(x - 3)^2 - 4$

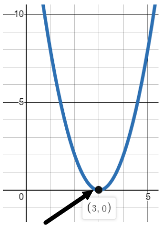
a & k have the same signs



0 solutions

$y = 2(x - 3)^2$

k = 0



1 solution

Finding the Vertex

There are two ways to do this:

1. Complete the square.

2. Find the x-intercepts (factor or quadratic formula). From there find "h" [(r+s)÷2] and then sub this into the equation to find y (the k value) **OR** find h using h = -b/2a and then sub in.

Example $y = 2x^2 + 5x + 2$

$y = 2(x^2 + 2.5x) + 2$

$y = 2(x^2 + 2.5x + (1.25)^2 - (1.25)^2) + 2$

$y = 2(x^2 + 2.5x (1.25)^2) - (2)(1.25)^2 + 2$

$y = 2(x + 1.25)^2 - 3.125 + 2$

$y = 2(x + 1.25)^2 - 1.125$ Vertex is (-1.25,-1.125)

$y = 2x^2 + 5x + 2$

$(a)(c) = 4 \xrightarrow{\quad} 4 \times 1$

$0 = 2x^2 + 4x + 1x + 2$

$0 = 2x(x + 2) + 1(x + 2)$

$0 = (x + 2)(2x + 1)$

$x + 2 = 0 \qquad 2x + 1 = 0$

$2x = -1$

$x = -2$ and $x = -0.5$

OR find h using $h = -b/2a$

$h = -5/2(2)$

$h = -1.25$

$$h = \frac{-2 + (-0.5)}{2}$$

$h = -1.25$

Sub into the equation

$y = 2(-1.25)^2 + 5(-1.25) + 2$

$y = 3.125 - 6.25 + 2$

$y = -1.125$

Vertex is (-1.25,-1.125)

Quadratic Transformations

We can only identify the transformations from vertex form.

$$y = a(k(x - d))^2 + c$$

$|a| > 1$ then there is a vertical stretch by a factor of "a"

$0 < |a| < 1$ there is a vertical compression by a factor of "a"

If a is negative there is a reflection in the x-axis

$|k| > 1$ then there is a horizontal compression by a factor of "1/k"

$0 < |k| < 1$ there is a horizontal stretch by a factor of "1/k"

If k is negative there is a reflection in the y-axis

If d is positive there is a horizontal translation "d" units to the right (towards the positive numbers)

If d is negative there is a horizontal translation "d" units to the left (towards the negative numbers)

Recall: We must take d out from the bracket first and it will change sign

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If c is positive there is a vertical translation "c" units up

If c is negative there is a vertical translation of "c" units down

Examples

State the transformations to the following: $y = -3(-2x - 8)^2 + 1$

Always state the stretches/compressions first along with any reflections. Remember that you may need to take out a factor of "k" from the bracket to reveal the correct horizontal translation.Then state the translations.

$$y = -3(-2x - 8)^2 + 1$$

$$y = -3(-2(x + 4))^2 + 1$$

Reflection in the x-axis (a is negative)

Vertical stretch by a factor of 3 (a is 3)

Reflection in the y-axis (k is negative)

Horizontal compression by a factor of 1/2 (k is 2)

Horizontal translation of 4 units left (d is -4)

Vertical translation of 1 unit up (c is 1)

Graphing a Quadratic Using Transformations

Example: Graph the equation $y = 2(x - 1)^2 - 3$

Start by plotting the vertex (1,-3)

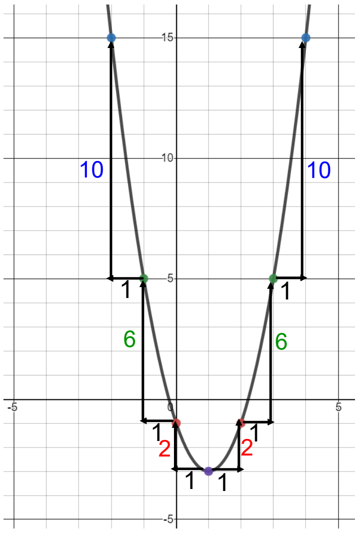
Next multiply (1,3,5) by the "a" value

$\longrightarrow 2(1,3,5) = (2,6,10)$

From the vertex move left one (1/k) and then up 2, then from there left one (1/k) and then up 6, and then from there left one (1/k) and up 10.

Repeat the process but this time move right from the vertex and move right each time instead of left.

If these step numbers are negative then you would go down instead of up.



To find where a point (x,y) moves to use:

$x: \longrightarrow x/k + d$

$y: \longrightarrow a(y) + c$

Kraft Dinner and Cheese

Applies to x-value Applies to y-value

Writing the Equation of a Quadratic

Depending upon the information given we can write it one of two ways.

If given the x-intercepts and a point we can write it in factored form $y = a(x - r)(x - s)$

If given the vertex and a point we can write it in vertex form $y = a(x - h)^2 + k$

1. Decide which form you are using
2. Sub in the information you know
3. Solve for the "a" value
4. Write the equation

Examples

1. Vertex at (-4,8) and the y-intercept at (0,10)

$(h,k) = (-4,8)$ and $(x,y) = (0,10)$

$$y = a(x - h)^2 + k$$

$$10 = a(0 - (-4))^2 + 8$$

$$2 = a(4)^2$$

$$\frac{2}{16} = \frac{a(16)}{16}$$

$$\frac{1}{8} = a$$

$$y = \frac{1}{8}(x + 4)^2 + 8$$

2. Roots at (2,0) and (-6,0) with a point (-1,4) on the parabola

$r = 2$, $s = -6$, and $(x,y) = (-1,4)$

$$y = a(x - r)(x - s)$$

$$4 = a((-1) - 2)((-1) - (-6))$$

$$4 = a(-3)(5)$$

$$\frac{4}{-15} = \frac{a(-15)}{-15}$$

$$\frac{-4}{15} = a$$

$$y = \frac{-4}{15}(x - 2)(x + 6)$$