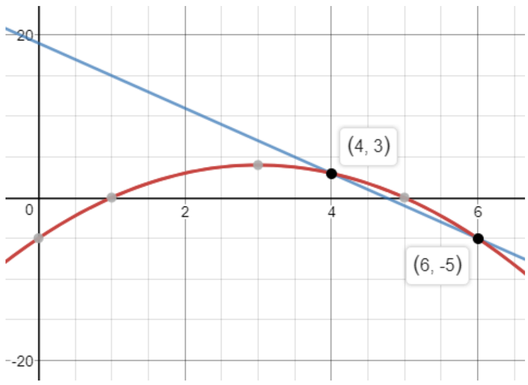
Solutions

Nov 20-18:35

2. Determine the point(s) of intersection algebraically.

a) $f(x) = -x^2 + 6x - 5$, $g(x) = -4x + 19$



$-x^2 + 6x - 5 = -4x + 19$

$0 = x^2 - 6x - 4x + 5 + 19$

$0 = x^2 - 10x + 24$

$0 = (x-4)(x-6)$

$\Rightarrow x = 4$ or $x = 6$

$y = -4(4) + 19$

$y = -16 + 19$

$y = 3$

$y = -4(6) + 19$

$y = -24 + 19$

$y = -5$

$\Rightarrow (4, 3)$ and $(6, -5)$

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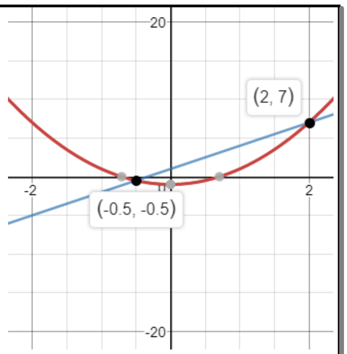
b) $f(x) = 2x^2 - 1, g(x) = 3x + 1$



$2x^2 - 1 = 3x + 1$

$2x^2 - 3x - 1 - 1 = 0$

$2x^2 - 3x - 2 = 0$

$ac = 2(-2) = -4$

$-4 \times 1 = -4$

$-4 + 1 = -3$

$2x^2 - 4x + x - 2 = 0$

$2x(x-2) + 1(x-2) = 0$

$(x-2)(2x+1) = 0$

$x - 2 = 0 \qquad 2x + 1 = 0$

$x = 2 \qquad \frac{2x}{2} = \frac{-1}{2}$

$x = -\frac{1}{2}$

$y = 3x + 1 \qquad y = 3x + 1$

$y = 3(2) + 1 \qquad y = 3(-\frac{1}{2}) + 1$

$y = 7 \qquad y = -\frac{1}{2}$

$\Rightarrow (2, 7)$ and $(-\frac{1}{2}, -\frac{1}{2})$

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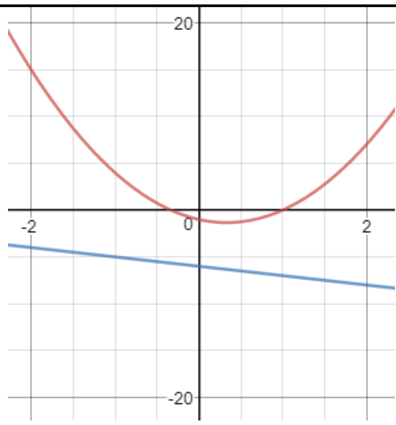
c) $f(x) = 3x^2 - 2x - 1, g(x) = -x - 6$



$3x^2 - 2x - 1 = -x - 6$

$3x^2 - 2x + x - 1 + 6 = 0$

$3x^2 - x + 5 = 0$

$x = \frac{1 \pm \sqrt{(-1)^2 - 4(3)(5)}}{2(3)}$

$x = \frac{1 \pm \sqrt{-59}}{6} \leftarrow$ no solution

$\Rightarrow$ no points of intersection

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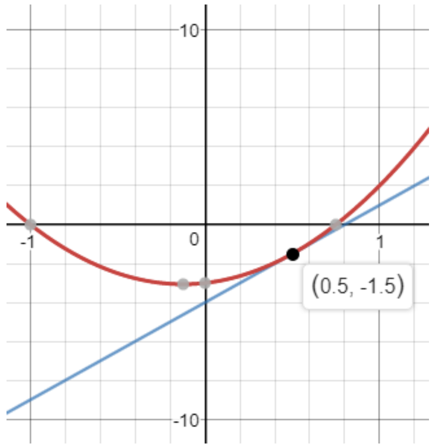
3. Determine the number of points of intersection of $f(x) = 4x^2 + x - 3$ and $g(x) = 5x - 4$ without solving.



$4x^2 + x - 3 = 5x - 4$

$4x^2 + x - 5x - 3 + 4 = 0$

$4x^2 - 4x + 1 = 0$

$b^2 - 4ac$

$= (-4)^2 - 4(4)(1)$

$= 16 - 16$

$= 0$

$\Rightarrow$ one point of intersection

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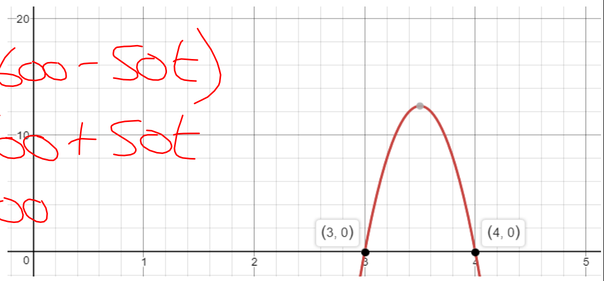
6. The revenue function for a production by a theatre group is $R(t) = -50t^2 + 300t$, where t is the ticket price in dollars. The cost function for the production is $C(t) = 600 - 50t$. Determine the ticket price that will allow the production to break even.

Profit = Revenue - Costs



Profit $= -50t^2 + 300t - (600 - 50t)$

$0 = -50t^2 + 300t - 600 + 50t$

$0 = -50t^2 + 350t - 600$

$0 = t^2 - 7t + 12$

$0 = (t-3)(t-4)$

$t - 3 = 0$ $\quad$ $t - 4 = 0$

$t = 3$ $\quad$ $t = 4$

$\Rightarrow$ Price of \$3 allows production to break even

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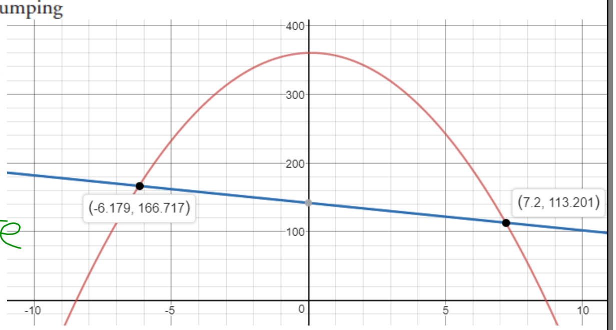
10. A daredevil jumps off the CN Tower and falls freely for several seconds before releasing his parachute. His height, $h(t)$, in metres, t seconds after jumping can be modelled by

$h_1(t) = -4.9t^2 + t + 360$ before he released his parachute; and
$h_2(t) = -4t + 142$ after he released his parachute.

How long after jumping did the daredevil release his parachute?



Looking for the time when the 2 functions are equal

$-4.9t^2 + t + 360 = -4t + 142$

$-4.9t^2 + 5t + 218 = 0$

$t = \dfrac{-5 \pm \sqrt{(5)^2 - 4(-4.9)(218)}}{2(-4.9)}$

$t = \dfrac{-5 \pm \sqrt{4297.8}}{-9.8}$

$t = -6.18$ or 7.20 sec

extraneous

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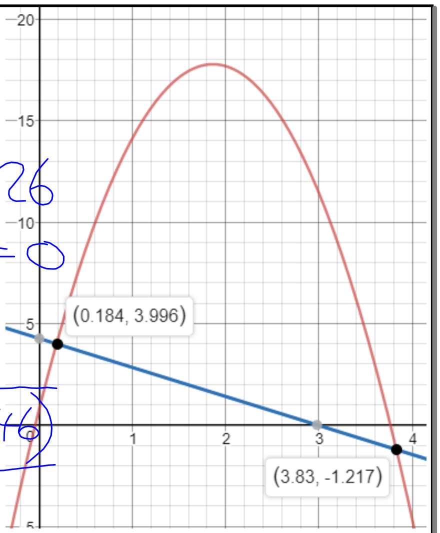
12. A punter kicks a football. Its height, $h(t)$, in metres, t seconds after the kick is given by the equation $h(t) = -4.9t^2 + 18.24t + 0.8$. The height of an approaching blocker's hands is modelled by the equation $g(t) = -1.43t + 4.26$, using the same t. Can the blocker knock down the punt? If so, at what point will it happen?



$-4.9t^2 + 18.24t + 0.8 = -1.43t + 4.26$

$-4.9t^2 + 18.24t + 1.43t + 0.8 - 4.26 = 0$

$-4.9t^2 + 19.67t - 3.46 = 0$

$t = \dfrac{-19.67 \pm \sqrt{(19.67)^2 - 4(-4.9)(-3.46)}}{2(-4.9)}$

$t = \dfrac{-19.67 \pm \sqrt{319.0929}}{-9.8}$

$t = 0.184$ or 3.830 seconds

extraneous

$\Rightarrow$ Blocker can block the punt after 0.184 sec

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