

Warm Up

A highway overpass has a shape that can be modelled by the equation of a parabola. If the edge of the highway is the origin and the highway is 10m wide, what is the equation of the parabola if the height of the overpass 2m from the edge of the highway is 13m?

Zeros at 0 and 10 \Rightarrow write equation in factored form
we know the point (2,13)

$$y = a(x-r)(x-s)$$

$$\Rightarrow 13 = a(2-0)(2-10)$$

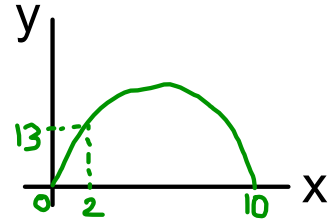
$$13 = a(2)(-8)$$

$$-\frac{13}{16} = \frac{-16a}{-16}$$

$$a = -\frac{13}{16}$$

$$\Rightarrow y = -\frac{13}{16}(x-0)(x-10)$$

$$y = -\frac{13}{16}x(x-10)$$



Linear-Quadratic Systems

Lesson objectives

- I know how to use substitution or elimination to find the solutions to a linear-quadratic system
- I know how to evaluate the two solutions to determine if they are both admissible

1.1

Lesson objectives

Teachers' notes

Lesson notes

Nelson Page 198 #s 2, 3, 6, 10 & 12

Linear-Quadratic Systems

In Grade 10 you studied linear systems - which meant determining where two lines crossed.

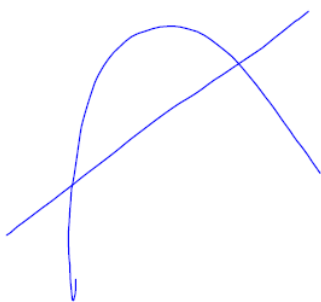
We used elimination and/or substitution to determine the solutions.

With quadratic-linear systems we usually use substitution since elimination gets tricky with no x^2 term in the linear equation.

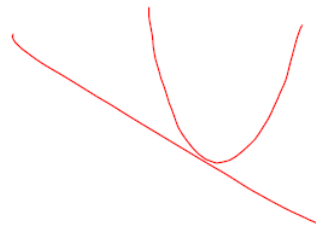


Number of Solutions

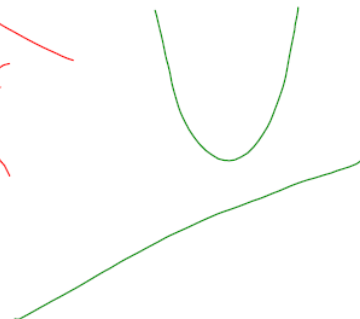
How many ways can a line and a parabola intersect?



2 points of intersection



1 point of intersection



0 points of intersection



Example

Determine the solutions

$$y = x^2 - x - 6 \text{ and } y = 2x - 2$$

Both "right hand sides" (RHS) = y , so they must be equal to each other

$$\Rightarrow x^2 - x - 6 = 2x - 2$$

$$x^2 - x - 2x - 6 + 2 = 0$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$\Rightarrow x = 4 \text{ or } -1$$

Now find y values for each x value

$$y = 2(4) - 2$$

$$y = 8 - 2$$

$$y = 6$$

$$y = 2(-1) - 2$$

$$y = -2 - 2$$

$$y = -4$$

$$\Rightarrow (4, 6) \text{ and } (-1, -4)$$

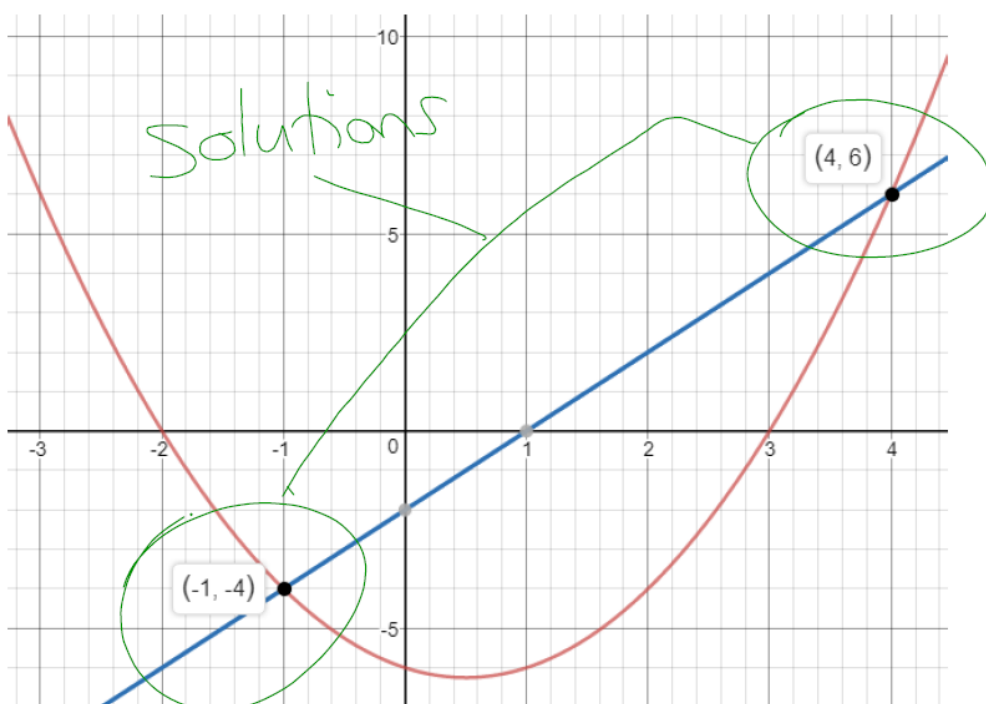
[Points of intersection]

Example

Determine the solutions

$$y = x^2 - x - 6 \text{ and } y = 2x - 2$$

This is the graph of the previous example



Example

A fish jumps over a waterfall following the path of $h_1(t) = -2t^2 + 4t + 6$, where t is in seconds and $h(t)$ is in metres above the water. If a bird is diving for the water on the path $h_2(t) = -6t + 14$, would the bird catch the fish? If so, at what height?

$$-2t^2 + 4t + 6 = -6t + 14$$

$$-2t^2 + 4t + 6t + 6 - 14 = 0$$

$$-2t^2 + 10t - 8 = 0$$

Common factor -2

$$t^2 - 5t + 4 = 0$$

$$(t-4)(t-1) = 0$$

$$\Rightarrow t = 4, t = 1$$

Find the heights

$$h = -6(4) + 14 \quad h = -6(1) + 14$$

$$h = -24 + 14 \quad h = -6 + 14$$

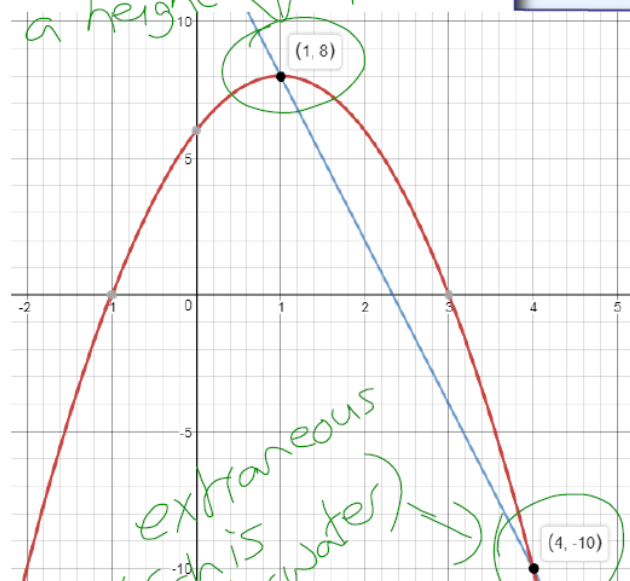
$$h = -10 \quad h = 8$$

Points of intersection are $(1, 8)$ and $(4, -10)$
Catches fish at height 8m

Example

A fish jumps over a waterfall following the path of $h_1(t) = -2t^2 + 4t + 6$, where t is in seconds and $h(t)$ is in metres above the water. If a bird is diving for the water on the path $h_2(t) = -6t + 14$, would the bird catch the fish? If so, at what height?

Solution. Catches fish after 1 second at a height of 8m



extraneous (fish is underwater)

