Solutions

Page 42 #s 1, 2, 3, 6ace, 7ad, 11abd, 12aceg

Page 44 # 16

1. State three numbers between 0 and 1.

Anything starting "0." or $\frac{\square}{\square}$ Small big

2. State four rational numbers between 3 and 4.

Anything starting "3." or $3\frac{\square}{\square}$ Small big

3. List all of the natural numbers from 1 through 5. Is this task easy to complete? Why or why not?

1, 2, 3, 4, 5

Easy. There are only 5 values in this FINITE SET.

6. a) How many natural numbers are there from 1 through 10?

c) How many rational numbers are there from 1 through 10?

e) How many real numbers are there from 1 through 10?

a) 10 numbers (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)

c) Infinite. We can always squeeze another element into this set.

e) Same as c). Infinite with the same reasoning.

7. State which set is denser (has higher density).

a) The set of whole numbers between 0 and 30 or the set of rational numbers between 0 and 30.

d) The set of even numbers between -50 and -20 or the set of irrational numbers between -50 and -20.

a) Whole numbers from 0 to 30 is finite. Rational numbers from 0 to 30 is infinite. ⇒ Rational numbers are denser.

d) Even numbers from −50 to −20 is finite. Irrational numbers from −50 to −20 is infinite.
⇒ Irrational numbers are denser.

11. The following sequences of numbers each have a *limit*. That is, they gradually get closer and closer to a specific number, called the *limit*. Identify the limit of each of the following sequences.

a) $7.1, 7.01, 7.001, 7.0001, \ldots$ b) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots$ d) $0.3, 0.33, 0.333, 0.3333, \ldots$

a) The limit is 7

b) The limit is 0

c) The limit is $\frac{1}{3}$

Recall: $\frac{1}{3} = 0.33333\ldots$ "the 3 continues"

13. For each of the following expressions, imagine the value of n getting higher and higher. What happens to the value of the expression as n approaches infinity?

a) n c) n^2 e) $\frac{1}{n^2}$ g) $\frac{n}{n^2-5}$

a) $n \to \infty$

$\Rightarrow n \to +\infty$

b) $n \to \infty$

$\Rightarrow n^2 \to +\infty$

e) $n \to \infty$

$\Rightarrow \frac{1}{n^2} \to \frac{1}{\infty}$

$\to 0$

g) $n \to \infty$

Bottom becomes much bigger than the top

$\frac{n}{n^2-5} \to 0$

16. Did you know it is possible to add infinitely many values and get a result that is not infinity? Consider the following sum.

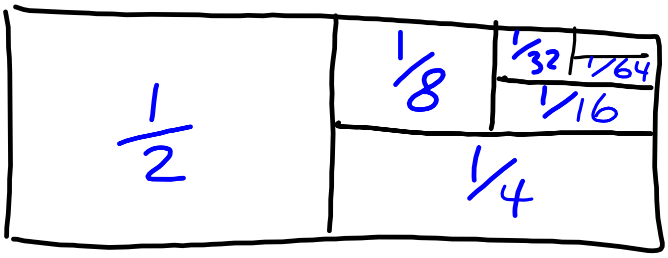
$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots$$

a) Hypothesize the value of this sum.

b) Use an area diagram to verify your hypothesis.

a) It like the blueberry pie question. This time we are adding up how much has been eaten. Limit $\longrightarrow 1$

b)



and so on...