

**Warm-up**

List **10** numbers between **2** and **3**.

All answers will start with 2. (...)

If this pattern continues, what value does it approach?

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$$

Numbers are getting progressively small, so the value will approach zero.

MTH1W Grade 9 Mathematics

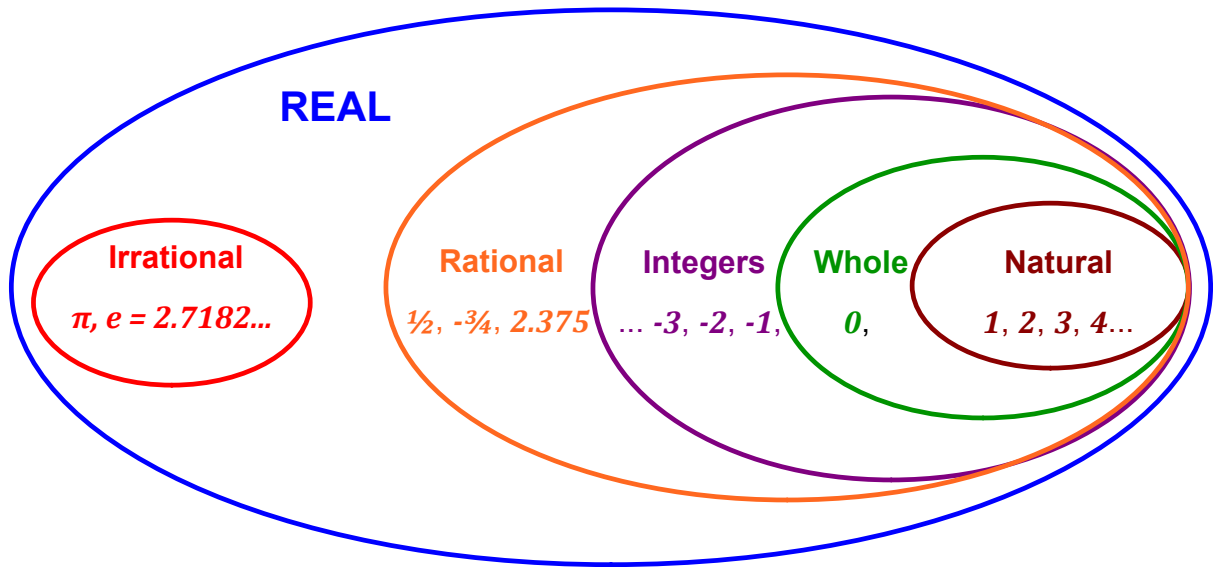
**1.6 Infinity, Limits, and Density**

- Goal(s)**
- Describe how various subsets of a number system are defined and describe similarities and differences between these subsets.
  - Use patterns and number relationships to explain density, infinity, and limits.

Page 42 #s 1, 2, 3, 6ace, 7ad, 11abd, 12aceg

Page 44 # 16

### Number Sets and Subsets

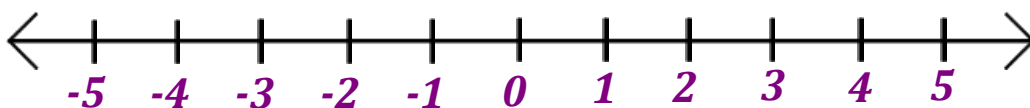


Recall: a subset is a set within another set.

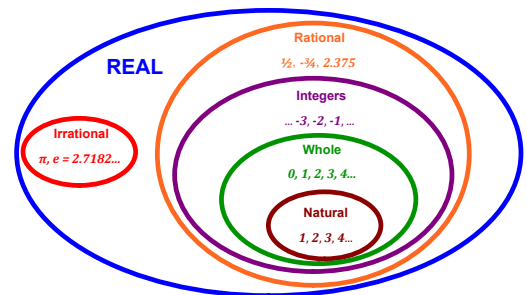
### Density of a Number Set

A number set is considered dense if you can always find another number between any two numbers.

Consider the set of **Real** numbers... would it be considered dense?



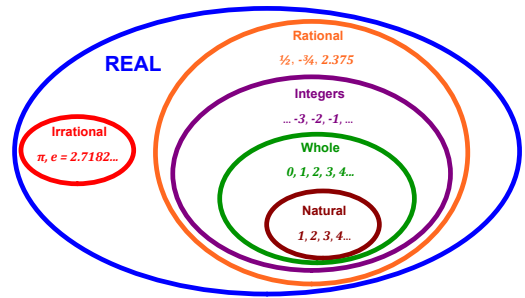
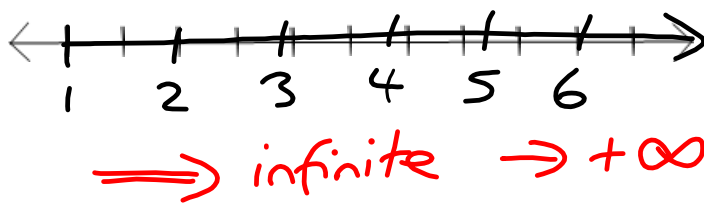
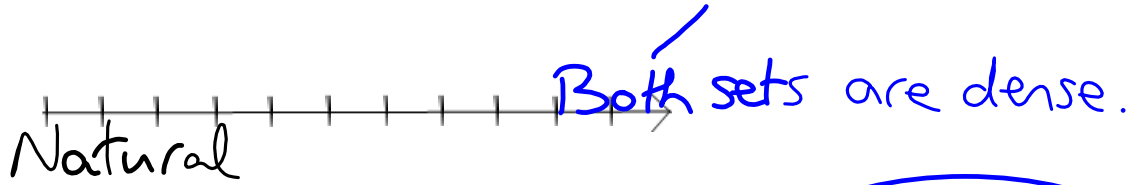
Yes! We can always find another value with an extra decimal place.



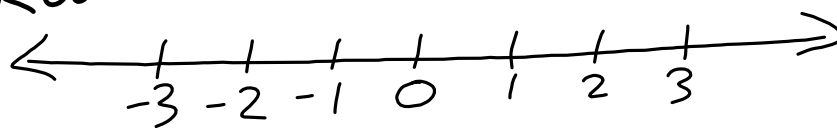
### Density of a Number Set

Compare the set of **natural** numbers and the set of **rational** numbers on two separate number lines.

Is each set finite or infinite? Is each set dense?



Rational

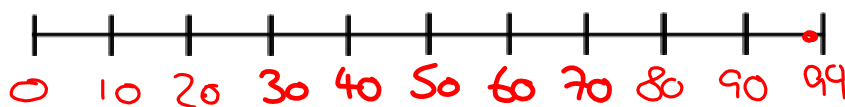


$\Rightarrow$  infinite  
 $\rightarrow -\infty$   
 $\rightarrow +\infty$

### Finite vs. Infinite

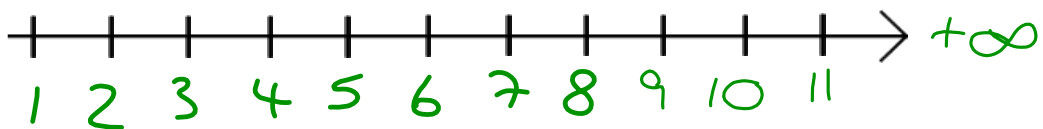
If something is **finite**, it has an **end**.

The set of whole numbers less than **100** is a finite set - *it begins at 0 and ends at 99.*



If something is **infinite**, it **goes on forever**.

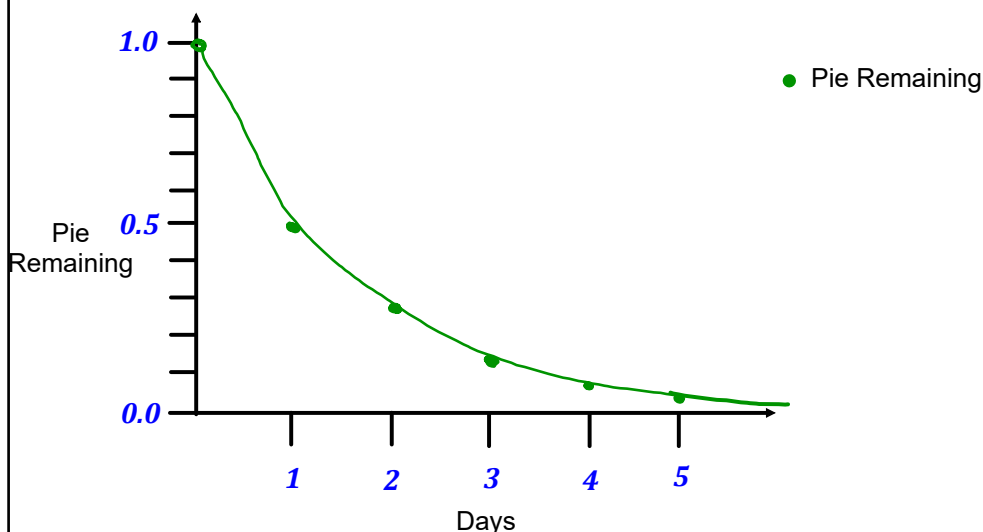
The set of natural numbers is an infinite set ...



Complete the table below showing how much pie is left each day compared to how much was eaten.

Day	Pie Remaining
1	$1 \times \frac{1}{2} = \frac{1}{2}$ or $0.5$
2	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ or $0.25$
3	$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ or $0.125$
4	$\frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$ or $0.0625$
5	$\frac{1}{16} \times \frac{1}{2} = \frac{1}{32}$ or $0.03125$

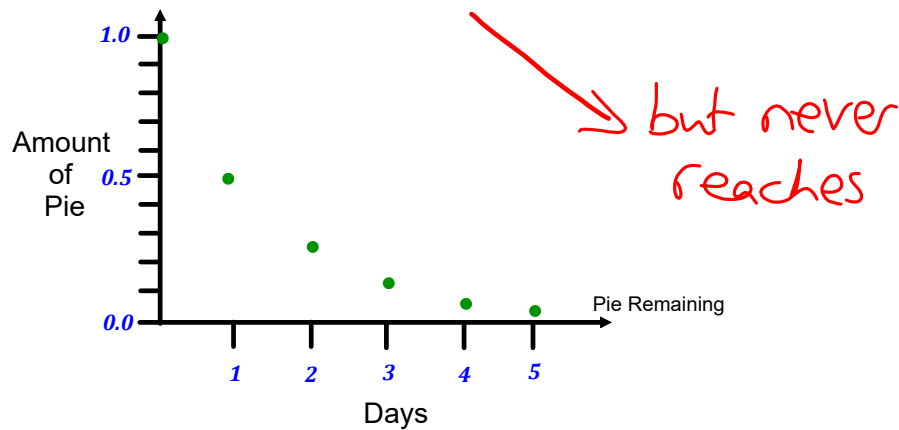
Use the data from the table to create a graph showing the amount of pie remaining over the five days.



If this pattern continues, will the pie ever be completely gone?

*No! You will always have something left if you only eat half of what you start with.*

A **limit** is a value that a graph approaches.



As the number of days increases, the fraction of pie remaining each day gets very small and approaches **0**.

*"The limit of the pie remaining each day approaches 0."*

A local charity is selling raffle tickets to win a gift card to a fancy restaurant. The odds of winning can be described by the equation  $\frac{1}{n}$ , where  $n$  represents the number of raffle tickets sold.

Show the probability of winning as the number of tickets sold increases. It may be helpful to represent the fractions as decimals.

Describe what happens to the probability of winning when a very large number of tickets is sold.

Number of Tickets Sold	Probability of Winning
1	$1 \div 1 = 1$
2	$1 \div 2 = 0.5$
5	$1 \div 5 = 0.2$
10	$1 \div 10 = 0.1$
100	$1 \div 100 = 0.01$
1 000	$1 \div 1000 = 0.001$
10 000	$1 \div 10000 = 0.0001$
100 000	$1 \div 100000 = 0.00001$

Probability of winning decreases when more tickets are sold.