Introduction to Probability

Extra Practice

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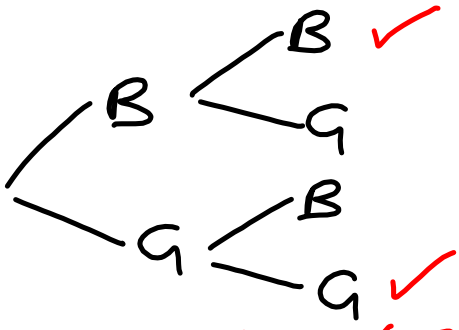
Solutions

1. A married couple decides to have two children. Assuming that they do, what is the probability that they will either have two boys or two girls?

A 0.125 B 0.25
C 0.5 D $0.\overline{6}$

Dependent events



$P(\text{2 Boys or 2 Girls}) = P(\text{2 Boys}) + P(\text{2 Girls})$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{2}{4}$

$= \frac{1}{2} \Rightarrow C$

2. Natalie logged on to a social media website 50 times. Fifteen of those times she encountered a pop-up advertisement. What is the experimental probability that Natalie will see a pop-up at this site?

A 7.5% B 15%
C 30% D 70%

$P(\text{Pop-up add}) = \frac{15}{50}$

$= \frac{3}{10}$

$= 30\% \Rightarrow C$

3. This spinner is spun 20 times and lands on green 5 times. Identify the true statement.

A The theoretical probability of landing on green is 20% and the experimental probability of landing on green is 20%.

B The theoretical probability of landing on green is 20% and the experimental probability of landing on green is 25%.

C The theoretical probability of landing on green is 25% and the experimental probability of landing on green is 20%.

D The theoretical probability of landing on green is 25% and the experimental probability of landing on green is 25%.

Theoretical

$P(\text{Green}) = \frac{1}{5}$ (20%)

Experimental

$P(\text{Green}) = \frac{5}{20} = \frac{1}{4}$ (25%)

$\Rightarrow$ B

4. A fair coin is flipped four times. What is the probability that it will land heads exactly once?

Four times $\Rightarrow 2 \times 2 \times 2 \times 2 = 16$ outcomes

Head on first flip, other three flips tails

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$

Head on second flip, other three flips tails

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$

Head on third flip, other three flips tails

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$

Head on fourth flip, other three flips tails

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$

$\Rightarrow \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{4}{16} = \frac{1}{4}$

5. Marlis feels 80% confident that she will pass her driver's exam.

a) What type of probability is Marlis using? Explain your choice.

Subjective — no data to justify this prediction, it is based on a feeling.

b) What are the odds in favour of Marlis passing her driver's exam, based on her probability estimate? Justify your reasoning.

80% pass, so it must be 20% fail

Odds in favour

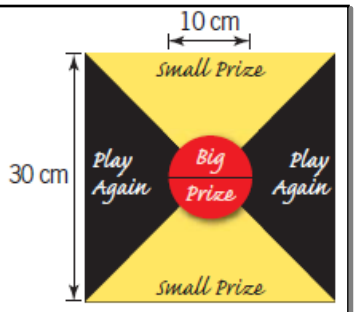
80 : 20

(÷ by 20) 4 : 1

6. Tenzin is playing a carnival game in which he throws a dart at the target shown below. Assuming that he is equally likely to hit any point on the target, what is the probability Tenzin wins the following on a given throw?

a) a big prize

b) a small prize



Total area $= 30 \times 30 = 900\text{cm}^2$

a) Area of big prize $= \pi r^2$

$= \pi\left(\frac{10}{2}\right)^2$

$= \pi(25)$

$= 25\pi\ \text{cm}^2$

$\Rightarrow P(\text{Big prize}) = \frac{25\pi}{900} = 8.7\%$

b) Area of small prize $= \frac{900 - 25\pi}{2}$

$= 410.73\ \text{cm}^2$

$\Rightarrow P(\text{Small prize}) = \frac{410.73}{900} = 45.6\%$

7. In a game involving two standard dice, you win if you roll a sum of 7 or 11, or if you roll doubles (both dice showing the same number).

a) What are the odds against you winning this game?

b) Explain how you solved this problem.

$P(\text{wining}) = P(7) + P(11) + P(\text{double})$

$= \frac{6}{36} + \frac{2}{36} + \frac{6}{36}$

$= \frac{14}{36}$

$= \frac{7}{18}$

a) $\frac{7}{18}$ wining $\Rightarrow$ $\frac{11}{18}$ not wining

Odds against wining

$\frac{11}{18} : \frac{7}{18}$

(x by 18) $11 : 7$

b) Identify the favourable outcomes (14). Identify the total number of outcomes (36). Express the probability of wining and hence not wining. Write the ratio and simplify.

8. Mr. Dobson's tie rack is shown below.

What is the probability that Mr. Dobson randomly selects

a) a solid blue tie or a polka dot tie?

b) a striped tie or a solid coloured tie?

c) a solid black tie or a striped tie?

d) a solid coloured tie or a solid blue tie?

a) $P(\text{SB or PD}) = P(\text{SB}) + P(\text{PD})$

$= \frac{1}{8} + \frac{1}{8}$

$= \frac{2}{8} = \frac{1}{4}$

b) $P(\text{ST or SC}) = P(\text{ST}) + P(\text{SC})$

$= \frac{4}{8} + \frac{3}{8}$

$= \frac{7}{8}$

c) $P(\text{SB or ST}) = P(\text{SB}) + P(\text{ST})$

$= \frac{1}{8} + \frac{4}{8}$

$= \frac{5}{8}$

d) $P(\text{SC or SB}) = P(\text{SC}) + P(\text{SB}) - P(\text{SC and SB})$

$= \frac{3}{8} + \frac{1}{8} - \frac{1}{8}$

$= \frac{3}{8}$

A pencil case contains 4 yellow, 3 green, and 2 pink highlighters. Without replacement, what is the probability that you select

a) A yellow, followed by a green

$P(YG) = P(Y) \times P(G|Y)$

$= \frac{4}{9} \times \frac{3}{8} = \frac{1}{6}$

b) A yellow, followed by a yellow, followed by a pink

$P(YYG) = P(Y) \times P(Y|Y) \times P(G|Y|Y)$

$= \frac{4}{9} \times \frac{3}{8} \times \frac{3}{7} = \frac{1}{14}$

c) A green, followed by a pink

$P(GP) = P(G) \times P(P|G)$

$= \frac{3}{9} \times \frac{2}{8} = \frac{1}{12}$

d) Not a yellow, followed by a yellow

$P(Y'Y) = P(Y') \times P(Y|Y')$

$= \frac{5}{9} \times \frac{4}{8} = \frac{5}{18}$